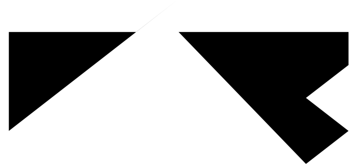


Figure 5.1. Basic dc-dc converter and associated voltage output

If the switch is always on, it is easy to see that the average output voltage will equal the input voltage; however, as the percentage of time that the switch is off increases, the average output voltage will decrease and will obviously become equal to zero when the switch is always off. The scheme thus far described is a buck converter (which has the ability to regulate the output voltage from a maximum value equal to the supply voltage to a minimum value equal to zero volts); however, it can be seen that while one is able to control the average value of the output voltage, the instantaneous voltage fluctuates between zero and V_1. This fluctuation is not acceptable in most of the applications where a regulated dc supply voltage is required.

The problem of output voltage fluctuation is largely solved by using a low pass filter consisting of a series inductor and a parallel capacitor. Figure 5.2 shows the schematic of the basic dc-dc converter with the low pass filter in place. It can also be seen that a diode

sawtooth voltage, the switch is turned off. If the dc-dc converter is to have closed loop feedback, then the output voltage would be monitored and compared with a desired output voltage (see Figure 5.4). The difference between the actual and the desired output voltage will regulate the magnitude of the dc control signal and will thus control the duty ratio of the system.

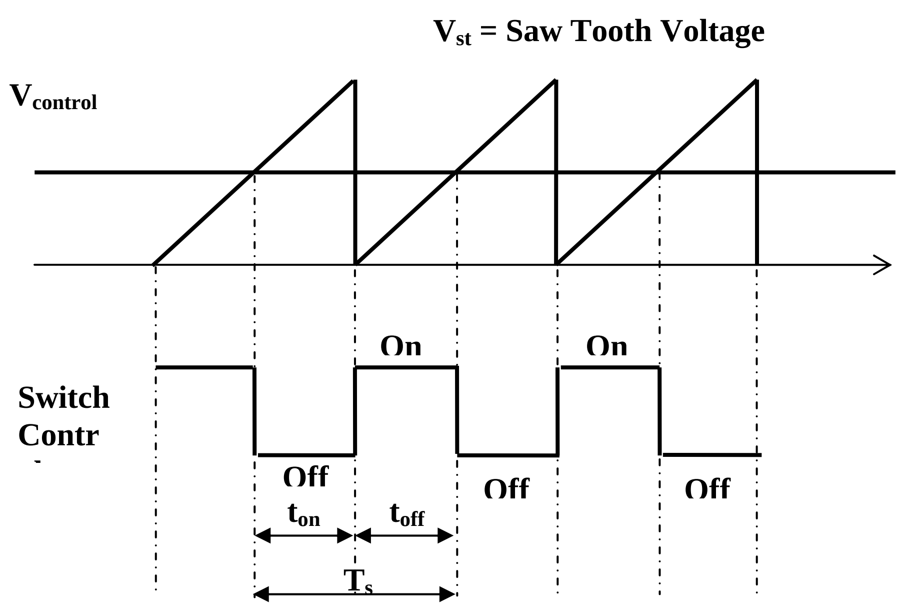


Figure 5.3. Control signal and sawtooth voltage waveforms and switching signal sent to turn switch on and off

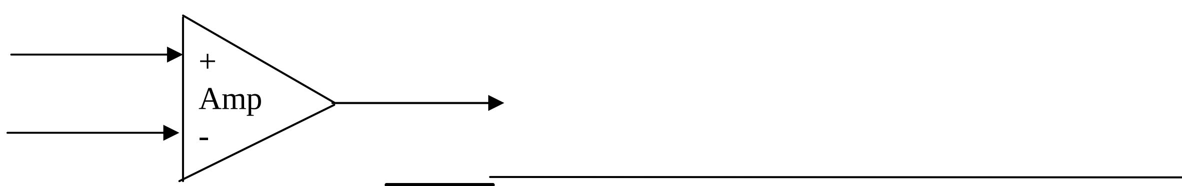


Figure 5.4. Schematic diagram of comparators and op-amp used to control signal of a converter

5.2 Derivation of a Buck Converter Operating in Steady State

The derivation for the steady state equivalent circuit of a buck converter builds upon the result found for the equivalent circuit which represents the rectifier feeding a resisti[redacted] load. Figure 5.5 shows the current flowing through the inductor L_p (see Figure 5.6) a[redacted] with the switching functions associated with the three modes of operation for th[redacted] converter. The first mode is when the transistor T1 is on, the second is when T1[redacted] the current flowing through the inductor L_p is greater than zero, and the third m[redacted]hen the transistor T1 is off and the current flowing through the inductor L_p is z[redacted]

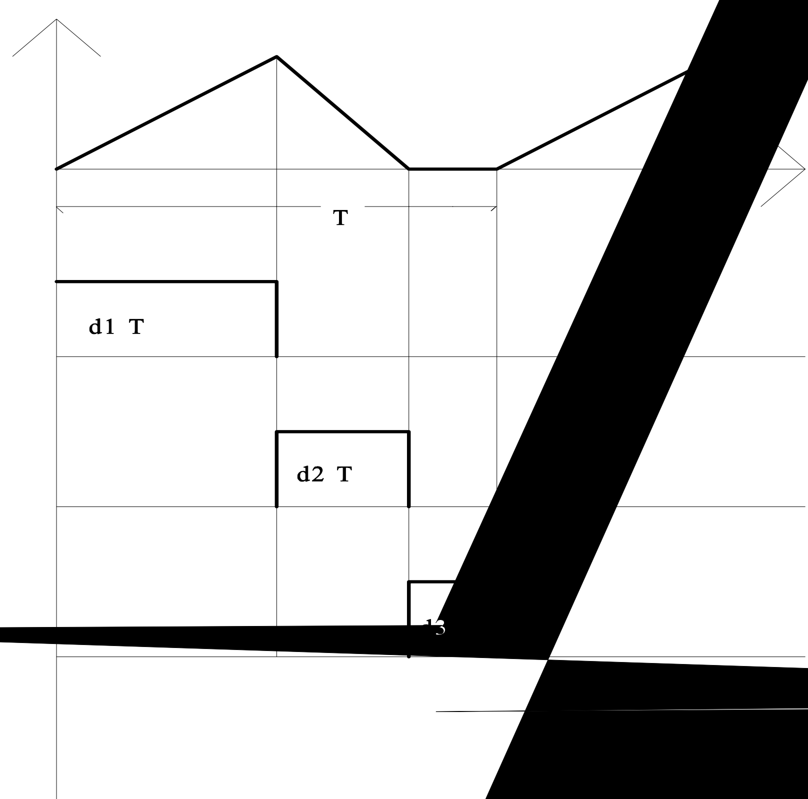


Figure 5.5. Inductor current and switching functio[redacted]

Before beginning the derivation for the buck converter, the mathematical basis for the fundamental approximation in the state-space averaging approach will be given. The derivation is taken from [35].

Let two linear systems described by

(i) Interval Td_1 , $0 < t < t_o$:

(ii) Interval Td_2 , $t_o < t < T$:

The exact solution of the state-space equations are where

$$e^{AT} = e^{d_2 A_2 T} e^{d_1 A_1 T} \ .$$

The first approximation solutions are

$$\begin{aligned} e^{d_1 A_1 T} &\quad I + d_1 A_1 T \\ e^{d_2 A_2 T} &\quad I + d_2 A_2 T \ , \end{aligned} \tag{5.6}$$

where I is the identity matrix.

Therefore,

$$e^{d_2 A_2 T} e^{d_1 A_1 T} \quad e^{(d_1 A_1 + d_2 A_2)T} \ , \tag{5.7}$$

which results in the approximate solution

$$x(T) \quad e^{(d_1 A_1 + d_2 A_2)T} x(0) \ . \tag{5.8}$$

This is the same as the solution of the following linear system equation for x(T):

$$\dot{x} = (d_1 A_1 + d_2 A_2) x \ . \tag{5.9}$$

This equation is the averaged model obtained from the switched models given in Equations (5.1) and (5.2) .

The differential equations for each mode of the buck converter are:

Mode 1: S_1 T1 ON 0 t d_1T

For mode 1, shown in Figure 5.7, the systems equations can be written as

$$
\begin{aligned}
L_d \frac{dI_1}{dt} &= V_s - V_c1 \\
C_1 \frac{dV_c1}{dt} &= I_1 - I_p \\
L_p \frac{dI_p}{dt} &= V_c1 - V_{co} \\
C_o \frac{dV_{co}}{dt} &= I_p - \frac{V_{co}}{R_L} \; .
\end{aligned}
\tag{5.10}
$$

Equation (5.10) can be written in matrix representation as

$$
\frac{dx}{dt} = [\ A_1\ ]\ x + [\ B_1\ ]\ u \ , \tag{5.11}
$$

where

$$
x = \begin{bmatrix} I \\ V \end{bmatrix}, A = \begin{bmatrix} 0 & -\frac{1}{L} & 0 & 0 \\ \frac{1}{C} & -\frac{1}{C} & 0 & 0 \\ 0 & \frac{1}{L} & 0 & -\frac{1}{L} \\ 0 & 0 & \frac{1}{C} & -\frac{1}{C\ R} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \end{bmatrix}, u = V \ .
$$

Mode 2: S_2 T1 OFF (I_p >0) d_1T t d_2T

For mode 2, shown in Figure 5.8, the systems equations can be written as

$$
\begin{aligned}
L_d \frac{dI_1}{dt} &= V_s - V_{c1} \\
C_1 \frac{dV_{c1}}{dt} &= I_1 \\
L_p \frac{dI_p}{dt} &= -V_{co} \\
C_o \frac{dV_{co}}{dt} &= I_p - \frac{V_{co}}{R_L} \ .
\end{aligned}
\tag{5.12}
$$

For mode 2, shown in Figure 5.8, the systems equations can be written as

$$
\frac{dx}{dt} = [\ A_2\ ]\ x + [\ B_2\ ]\ u \ , \tag{5.13}
$$

where

$$
x = \begin{bmatrix} I_1 \\ V_{c1} \\ I_p \\ V_{co} \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -\frac{1}{L_d} & 0 & 0 \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{L_p} \\ 0 & 0 & \frac{1}{C_o} & -\frac{1}{C_o R_L} \end{bmatrix}, \quad B_2 = \begin{bmatrix} \frac{1}{L_d} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad u = V_s \ .
$$

Now, using the result of Equation (5.9) , it is possible to obtain the single vector equation

$$\frac{dx}{dt} = A\,x + B\,u \ , \qquad (5.16)$$

where

$$A = A_1 S_1 + A_2 S_2 + A_3 S_3 \qquad B = B_1 S_1 + B_2 S_2 + B_3 S_3 \ .$$

Supposing that the quantities x, u, S_1 , S_2 , S_3, and u vary around their respective steady state values, then the following substitutions may be made

$$\begin{aligned} S_1 &= s_1 + s_1 \\ S_2 &= s_2 + s_2 \\ S_3 &= s_3 + s_3 \\ u &= u + \tilde{u} \\ x &= X + \tilde{x} \\ \frac{dx}{dt} &= \frac{\tilde{d}x}{dt} \ . \end{aligned}$$

Under these conditions, Equation (5.16) becomes

The last term can be ignored if the changes (perturbations) are much smaller than the corresponding steady-state values. The steady state waveforms can now be separated into dc and ac components. Of interest here is the dc component which is given as

$$[\, A_1 s_1 + A_2 s_2 + A_3 s_3 \,]\, X + [\, B_1 s_1 + B_2 s_2 + B_3 s_3 \,]\, u = 0 \ .$$

The dc (average value) of the switching functions are

$$\begin{aligned} s_1 &= \frac{d_1 T}{T} = d_1 \\ s_2 &= \frac{d_2 T}{T} = d_2 \\ s_3 &= \frac{d_3 T}{T} = d_3 \ , \end{aligned} \tag{5.19}$$

and, recognizing that

$$d_1 + d_2 + d_3 = 1 \ , \tag{5.20}$$

then d_3 may be written as

$$d_3 = 1 - d_1 - d_2 \ . \tag{5.21}$$

Substituting Equation (5.21) into (5.18) and rearranging gives

The effective input resistance is defined as the input voltage over the input current, i.e.

In terms of the duty cycle and the load resistance R_L the effective resistance can be found as follows:

In terms of d_1 and $d_{3,}$

when the buck converter is operating in mode one, the current i_p (t) rises linearly from a zero value at the beginning of the mode to a maximum value at time $t=d_1T$ of

The converter will then switch into mode two operation and the current will fall linearly from I_{pmax} to a zero value at a time $t=(d_1 + d_2)T$. The average value of the inductor current I_p over a complete cycle may be found by taking the area under the two triangles, and dividing by the total time T. Thus,

At steady state, the average current in the capacitor C_0 is zero. Since this is true, then

Substituting the value of V_{c1} in Equation (5.22) gives

$$V_{co} = R_L V_{co} \left(\frac{d_1 + d_2}{d_1} - 1 \right) d_1 T \frac{(d_1 + d_2)}{2 L_p} \ , \tag{5.30}$$

which, after simplification yields

$$d_1^2 d_2 + d_2^2 - \frac{2 L_p}{R_L} T = 0 \ . \tag{5.31-a}$$

In terms of d_1 and d_3 this equation becomes

Therefore, knowing the duty cycle d_1, the total period T, the load resistance R_L, and the value of the inductor L_p, the percentage of time that the converter is operating in discontinuous conduction mode may be determined by solving Equation (5.31-b) for d_3. If the solution for d_3 is either zero or negative, then the converter is always in continuous conduction mode.

For a given load resistance and period, the converter will tend toward discontinuous conduction mode as both d1 and L_p are decreased. At the boundary condition between continuous and discontinuous conduction mode, d_3=0, and Equation (5.31-b) may be written as

It can be seen that, as the duty cycle d_1 is decreased, the inductor value must be increased to satisfy the equation. As d_1 tends toward zero then the equation may be approximated as

Solving for L_p gives

Thus, if one wants to be sure of operating in continuous mode (which is normally the case because of the high stresses placed on the transistors when operating in discontinuous mode) regardless of the duty cycle, then a good ru resistance for the buck may be written as

$$R_{effbk} = \frac{R_L}{d_1^2} \quad . \tag{5.36}$$

The equivalent resistance given in Equation (5.36) may be substituted into Equation (4.31) to obtain the equivalent resistance of the buck-rectifier system. After the substitution, the equivalent resistance that the IPM (or any other power source) sees at its terminals is given in Equation (5.37) and will be used to predict the performance of the IPM feeding a rectifier-buck-resistance load.

$$R_{effbkrec} = \frac{R_L \quad {}^{2}}{12\, d_1^2} \tag{5.37}$$

5.2.1 Examination of Ideal Buck Converter

In order to gain an appreciation of the significance of Equation 5.36, various graphs are generated with the assumption that the dc voltage into the buck converter is a constant 10 volts dc, and the load resistance is a constant 10 ohms.

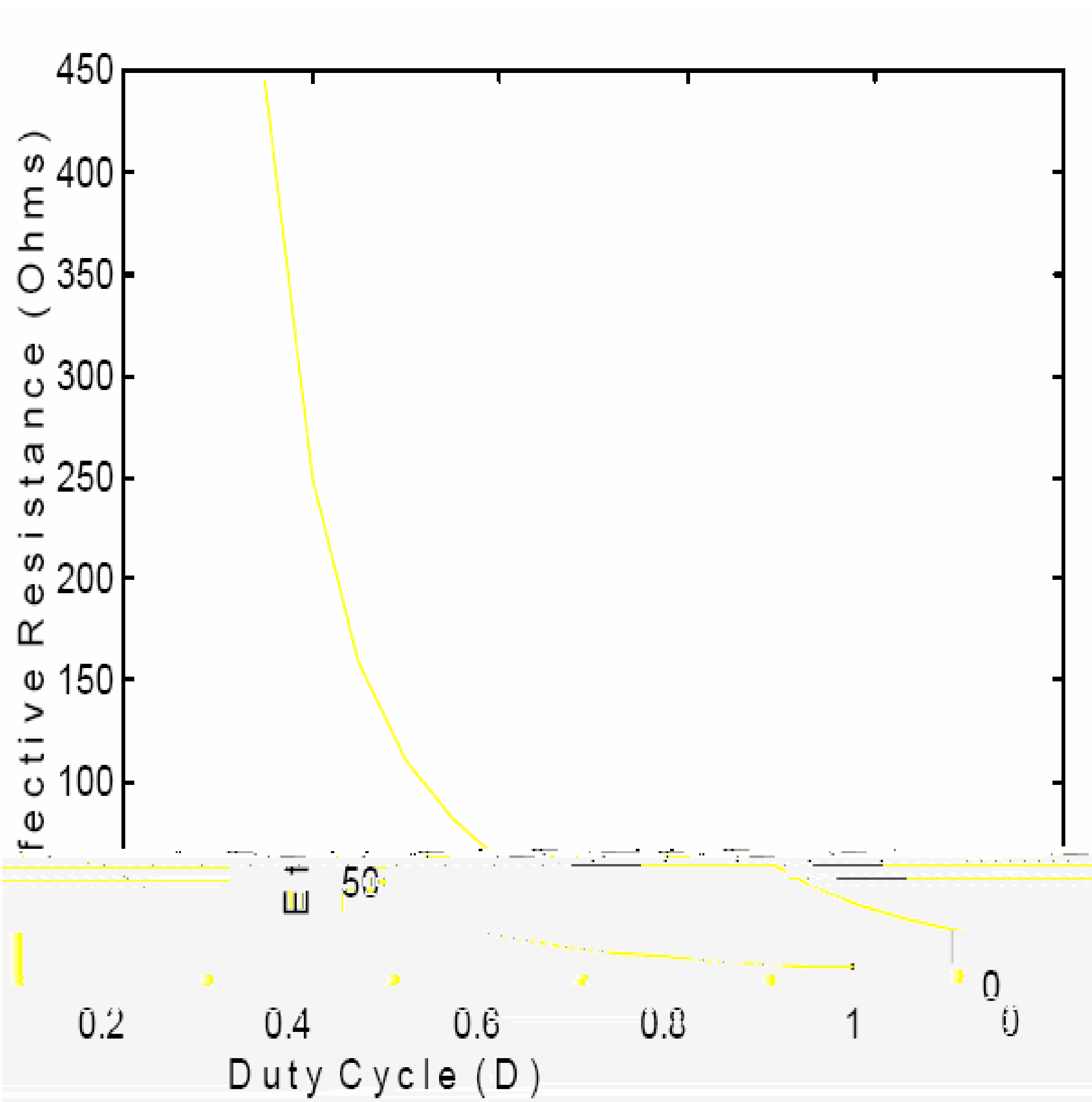


Figure 5.9. Effective resistance vs duty cycle for buck converter

Figure 5.9 displays how the effective resistance presented to the source decreases as the duty cycle increases. This trend is the same as that of the boost converter except for the fact that the buck effective resistance starts at an infinite resistance at a zero duty cycle and ends at the value of the load resistance, and the boost starts at the value of the load resistance and ends at a zero effective resistance.

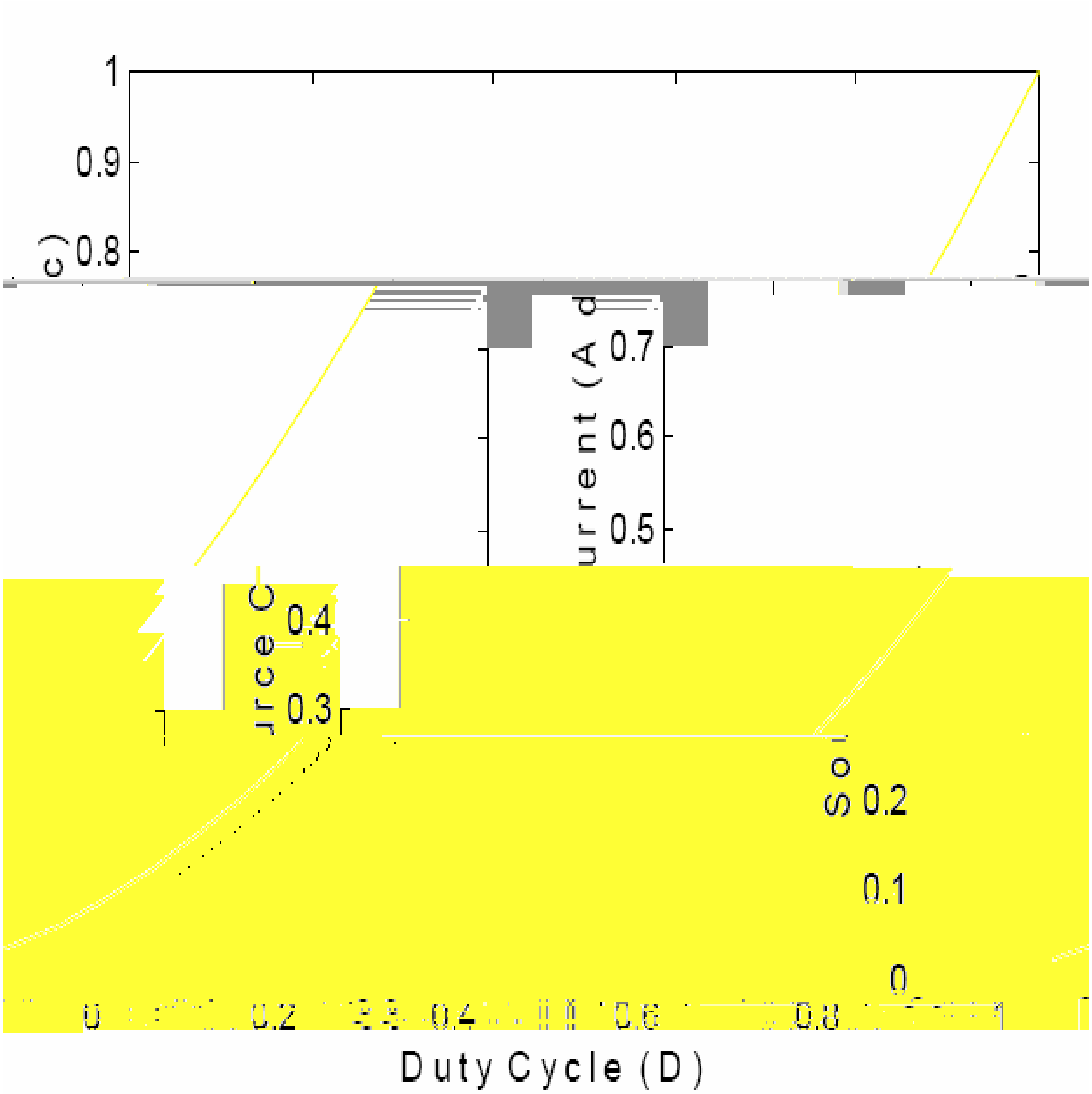


Figure 5.10. Source (input) current vs duty cycle for buck converter

Figure 5.10 shows how the source current increases as the duty cycle increases. At a duty cycle of zero the effective resistance is infinity and no current flows in the circuit. As the duty cycle increases, the source current increases exponentially, but only up a maximum value of V_{dc}/R_L.

Figure 5.11 shows how the rise of input power as the duty cycle is increased

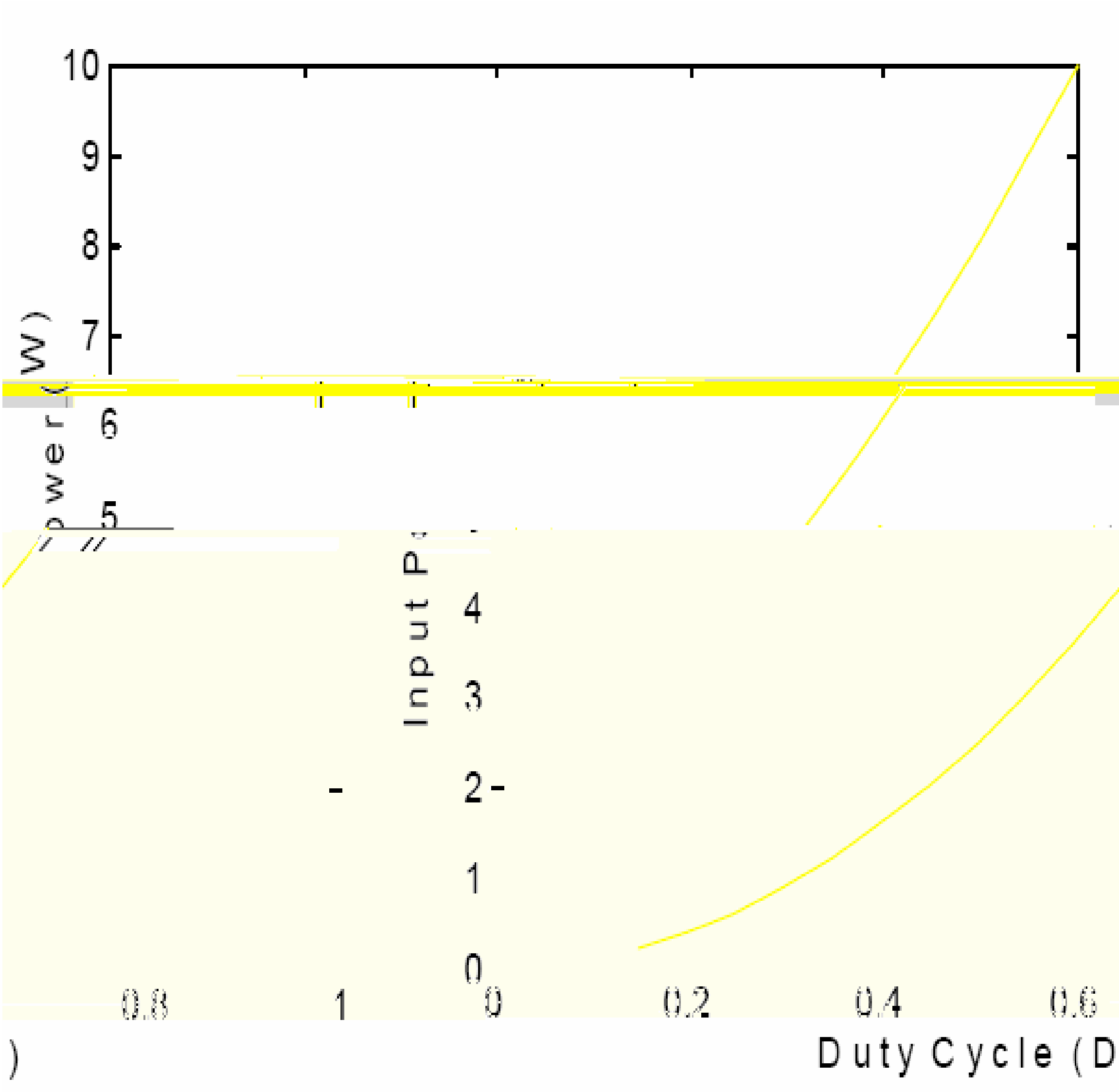


Figure 5.11 Input power vs duty cycle for buck converter.

5.3 Steady State Performance of an IPM Generator Feeding a Rectifier-Buck-Resistive Load

5.3.1 Introduction

In this section, the measured steady state performance of the IPM generator feeding a rectifier-buck-resistive load will be compared with the predicted performance of the system. In order to obtain a full performance curve (meaning that the performance of the IPM generator is tested for loads ranging from a light load to a large load) for the buck converter, it is not the load resistance R_L

5.3.2 Experimental and Predicted Performance Results

Figure 5.13 shows how measured and calculated line to neutral voltage of the generator varies as a function of the power out of the generator. If the rectifier-buck system truly appeared as a purely resistive load to the IPM, then the measured results would fall almost exactly on the calculated results line as they did in Figure 3.5.

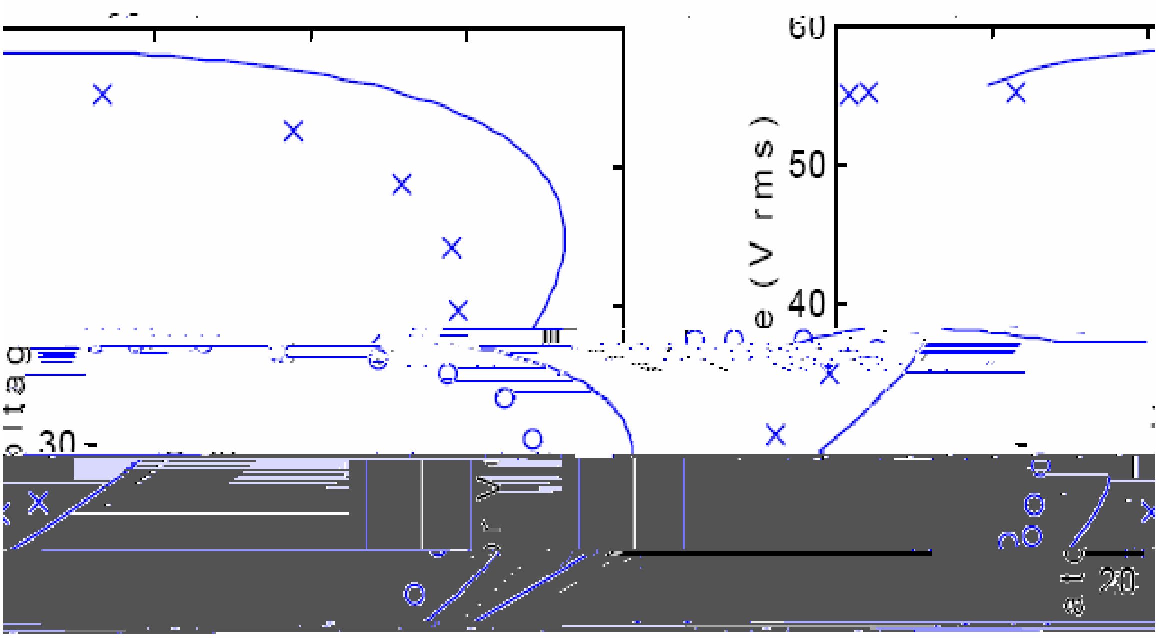


Figure 5.13. Measured and calculated generator line to neutral voltage vs generator output power for the IPM feeding a rectifier-buck-resistive load

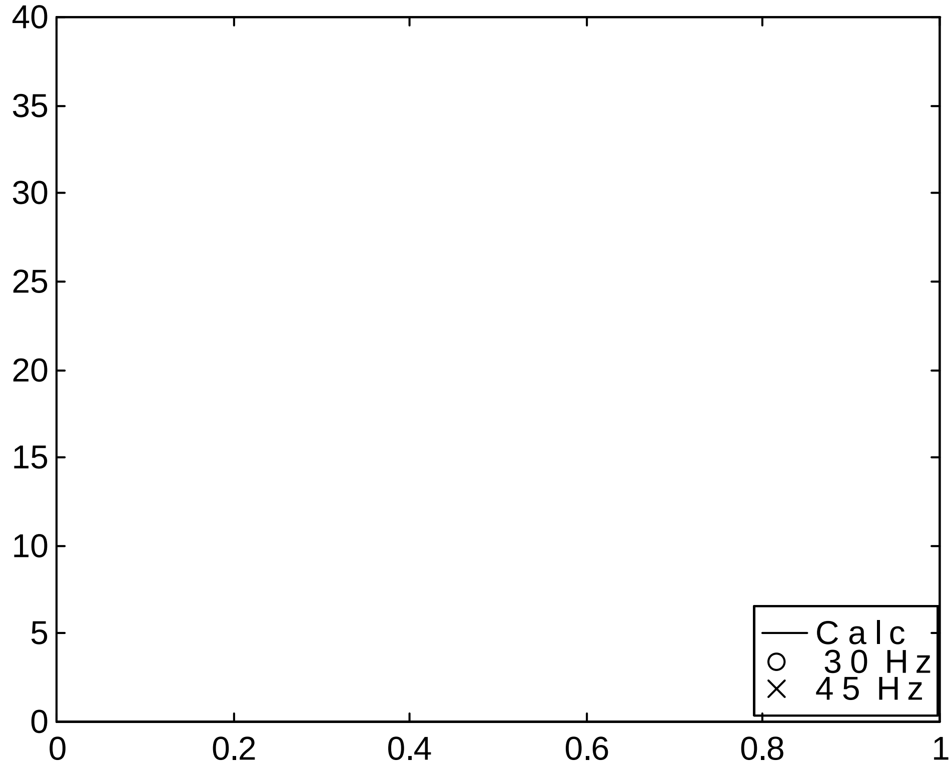

40
35
30
25
20
15
10
5
0
0
0.2
0.4
0.6
0.8
1
Calc
30 Hz
45 Hz

Figure 5.20. Measured power loss vs duty ratio for the IPM feeding a rectifier-buck-resistive load

5.4 Modeling of the Transistor

The modeling of a transistor in Simulink is similar to the model of the diode, except that the time the switch is on and the time in which the switch is off is controlled externally, and is not dependent on the voltage across or the current through the device. Figure 5.21 shows the Simulink block diagram of a UI transistor. In order for the transistor switch to be closed, the voltage across the transistor V_{ce} must be positive, and the control signal V_b

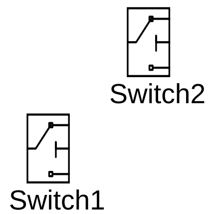


Figure 5.21. Simulink model of a UI transistor

Figure 5.22. Simulink model of an IU transistor

5.5 Comparison of Measured and Simulated Waveforms of IPM Machine Feeding a Rectifier- Buck-Resistive Load

This section includes the comparison between simulation and measured waveforms for the IPM generator feeding a rectifier-buck-resistive load. Two cases will be looked at. The first is when the buck converter is operating in continuous conduction mode, and the second is when the converter is operating in discontinuous conduction mode.

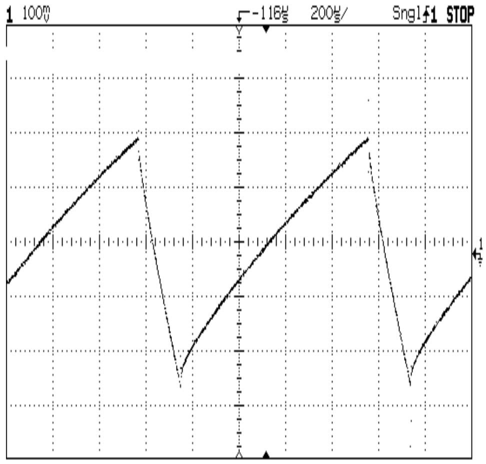
5.5.1 Buck Converter in Continuous Conduction Mode

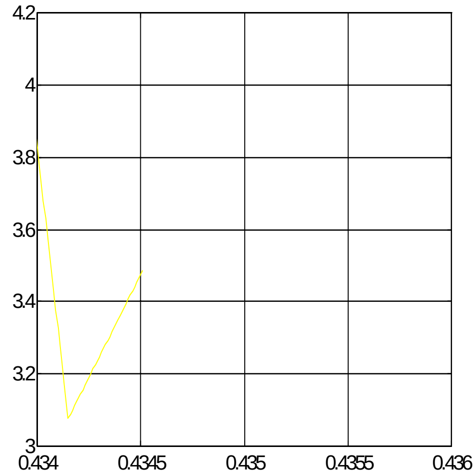
This section looks at measured and simulated waveforms when the IPM is feeding a rectifier-buck-resistive load topology for the case when the buck is operating in continuous conduction mode. The frequency of operation of the IPM machine is 30 Hz. The rectifier

Measurement Simulation

Figure 5.25. Measured and simulated line to line voltage waveforms for the generator feeding a rectifier-buck-resistive 10 resistive load. Rotor speed=900 rpm. Measured waveform scale: voltage: 50v/div, time 10ms/div

Figure 5.27 shows the measured and simulated current in the inductor L_p . It can be seen from the figure that, similar to the current I_p depicted in Figure 5.5, the current rises almost linearly when the transistor is turned on and falls linearly when the transistor is turned off; however, unlike the current in Figure 5.5, measured and simulated currents never



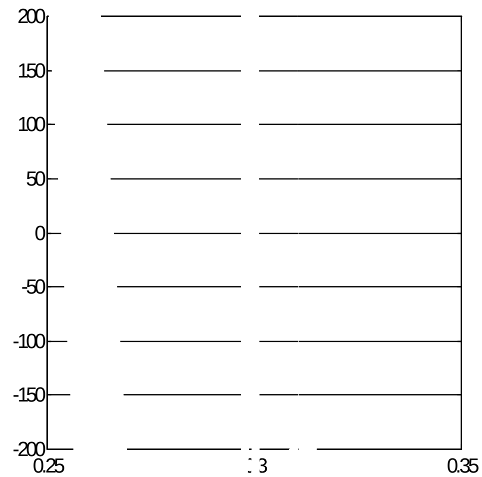


Measurement Simulation

Figure 5.26. Measured and simulated generator current waveforms for the generator feeding a rectifier-buck-resistive 10 resistive load. Rotor speed=900 rpm. Measured waveform scale: current: 1A/div, time 5ms/div

Measurement Simulation

Figure 5.27. Measured and simulated buck inductor current waveforms for the generator feeding a rectifier-buck-resistive 10 resistive load. Rotor speed=900 rpm. Measured waveform scale: current: .1A/div, time .2ms/div



ıent a ı

ːasured and simulated generator lin to t waveforms for the
g a rectifier-buck-resistive 10 r sistiv tor speed=900 rpm.
ɔrm scale: voltage: 50v/div, time s/div

.29 and 5.30 show the me imul e line voltage and line

ıerator. The simulat orm a ıch higher degree of

ion than the act rm e J is believed that the

that, since t ıake eff ive resistance which

ːes very ı for t ca of the rectifier, the

ːl d eratc pe ting under light load.

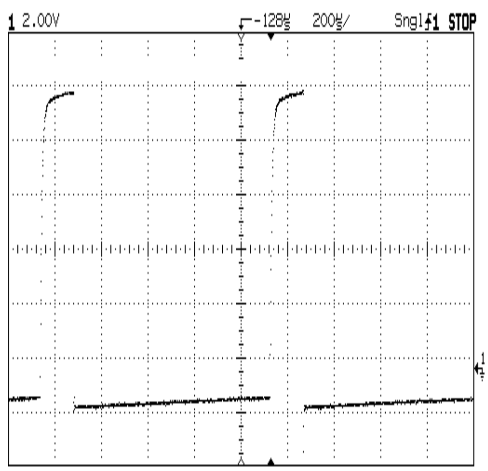
Measurement Simulation

Figure 5.30. Measured and simulated generator line current waveforms for the generator feeding a rectifier-buck-resistive 10 resistive load. Rotor speed=900 rpm. Measured genediffe waceein generen-15.97 T96D0.00006 T27 Tw006 6080 wavet wavein genemred

waveform.18.5(of re 5.3s0. 1 sim). 2 (whe w gene-15.197 033D0.0003 Tc50.0294 025[(waveconverter

Measurement Simulation

Figure 5.31. Measured and simulated inductor current waveforms for the generator feeding a rectifier-buck-resistive 10 resistive load. Rotor speed=900 rpm. Measured waveform scale: current: 5A/div, time .2ms/div



Measurement Simulation

Figure 5.32. Measured and simulated transistor current waveforms for the generator feeding a rectifier-buck-resistive 10 resistive load. Rotor speed=900 rpm. Measured waveform scale: current: 2A/div, time .2ms/div