# CHAPTER 3

# ANALYSIS OF THE INTERIOR PERMANENT MAGNET MACHINE FEEDING AN IMPEDANCE LOAD

## 3.1 Introduction

In this chapter, the IPM generator feeding an impedance load for the case when a shunt capacitor is present at the terminals, and also when the shunt capacitor is absent will be presented. A diagram of the topology is shown in Figure 3.1.

First, a general mathematical model of the IPM feeding an RL load will be developed using the dq synchronous reference frame transformation. The result of this derivation gives an eighth order equation. Then, the case when only a resistive load is fed by the IPM generator with shunt capacitor compensation will be examined. A fourth order equation is developed to describe the system. The final configuration, the IPM generator feeding a resistive load, will be examined and the mathematical model developed to describe the system results in a fourth order equation; however, it will be shown that the fourth order equation for this system can, in fact, be reduced to a simple quadratic equation by combining the stator resistance of the machine with the load resistance.

The comparison between experimental results and predicted results for the cases of a resistive load with and without shunt capacitor compensation will be made and the resulting graphs will be presented and commented on. The measured waveforms of the IPM feeding a resistive load will also be given in this chapter.

## 3.2 Mathematical Model for IPM Feeding an RLC Load

Figure 3.2 shows a schematic diagram of the IPM machine connected to an capacitive compensation

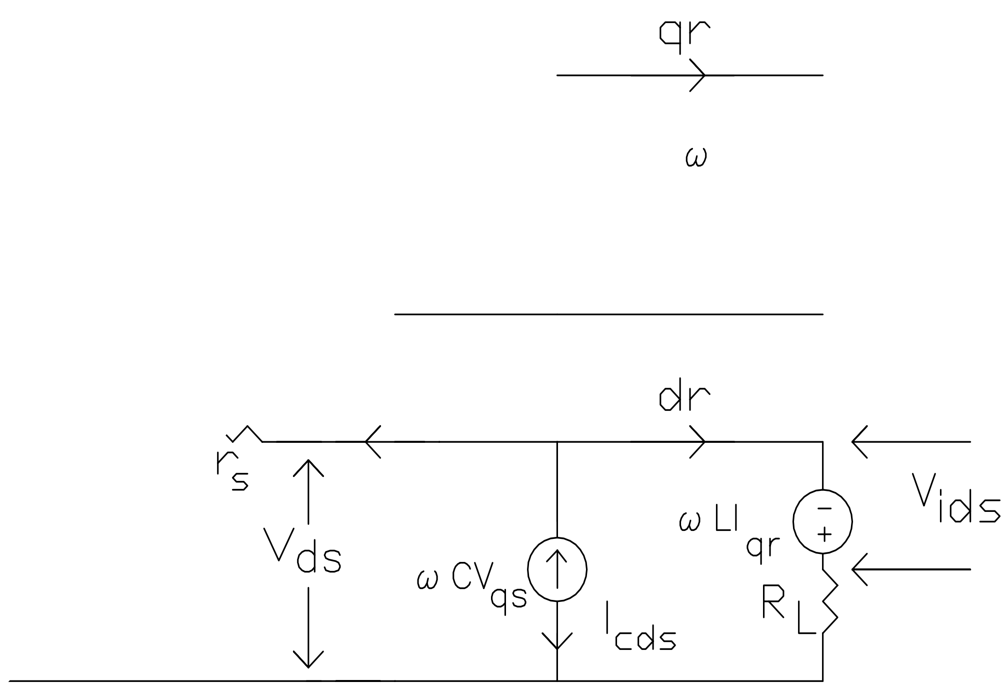



Figure 3.2. DQ equivalent circuit of IPM machine connected to an RLC load

$$i_{cqs} = \omega C\, v_{ds} + p\, C\, v_{qs}$$

$$i_{cds} = -\omega C\, v_{qs} + p\, C\, v_{ds}\ . \tag{3.4}$$

The equations given in Equations 3.1, 3.2, and3.4, at steady state, become, respectively,

$$V_{qs} = r_s\, I_{qs} + \omega\, \lambda_e + \omega\, L_d\, I_{ds}$$

$$V_{ds} = r_s\, I_{ds} - \omega\, L_q\, I_q\ ,$$

and

Substituting (3.3) into (3.6) and solving for the load currents gives

$$I_{qr} = \frac{V_{qs} R_L - \omega L V_{ds}}{R_L^2 + \omega^2 L^2}$$
$$I_{dr} = \frac{V_{qs} \omega L + R_L V_{ds}}{R_L^2 + \omega^2 L^2} . \tag{3.8}$$

Substituting the load current equations of (3.8) into (3.7) gives

$$V_{qs} = - \frac{( I_{qs} R_L^2 - I_{qs} \omega^2 L^2 - V_{qs} R_L )}{(L \omega - \omega C R_L^2 - \omega^3 L^2 C)}$$
$$V_{ds} = - \frac{( \omega C I_{qs} R_L^2 + \omega^3 C L^2 I_{qs} - I_{qs} \omega L + I_{ds} R_L )}{( \omega^4 L^2 C^2 - 2 \omega^2 C L + 1 + \omega^2 R_L^2 )} . \tag{3.9}$$

It is worthwhile to note that the above voltage equations of (3.9) were obtained using only the load side parameters. Therefore, these voltage equations can be used to equate to the terminal voltage of any electric machine (or transmission line) no matter what type of generator machine is used.

Now, for the permanent magnet side, as stated previously

$$V_{qs} = r_s I_{qs} + \omega \lambda_e + \omega L_d I_{ds}$$
$$V_{ds} = r_s I_{ds} - \omega L_q I_{qs} \; . \tag{3.10}$$

Therefore,

$$
\begin{aligned}
&-\frac{(I_{qs}\,R_L^2 - I_{qs}\,\omega^2\,L^2 - V_{qs}\,R_L)}{(L\,\omega - \omega\,C\,R_L^2 - \omega^3\,L^2\,C)} = r_s\,I_{qs} + \omega\,\lambda_e + \omega\,L_d\,I_{ds} \\
&-\frac{(\omega\,C\,I_{qs}\,R_L^2 + \omega^3\,C\,L^2\,I_{qs} - I_{qs}\,\omega\,L + I_{ds}\,R_L)}{(\omega^4\,L^2\,C^2 - 2\,\omega^2\,C\,L + 1 + \omega^2\,R_L^2)} = r_s\,I_{ds} - \omega\,L_q\,I_{qs}\ .
\end{aligned}
\tag{3.11}
$$

Manipulating (3.11) and solving for the currents gives

$$
I = \frac{n_{qq}1\,R_L^4 + n_{qq}2\,R_L^3 + n_{qq}3\,R_L^2 + n_{qq}4\,R_L + n_{qq}5}{d_d1\,R_L^4 + d_d2\,R_L^3 + d_d3\,R_L^2 + d_d4\,R_L + d_d5}
$$

$$
I_{ds} = \frac{n_{dd}1\,R_L^4 + n_{dd}2\,R_L^2 + n_{dd}3}{d_d1\,R_L^4 + d_d2\,R_L^3 + d_d3\,R_L^2 + d_d4\,R_L + d_d5}\ ,
$$

where

Now, the d and q axis currents can be combined using the following relationship:

anstituting4the valSub9, Substituting the values obtained in Equation (3.12) for $I_{ds}$ and $I_{ds}$

Where

$$A_8 = -n_{qq}1^2 - n_{dd}1^2 + I_s^2 d_d 1^2$$
$$A_7 = 2 I_s^2 d_d 1 d_d 2 - 2 n_{qq}1 n_{qq}2$$
$$A_6 = -2 n_{dd}1 n_{dd}2 - 2 n_{qq}1 n_{qq}3 + 2 I_s^2 d_d 1 d_d 3 + I_s^2 d_d 2^2 - n_{qq}2^2$$
$$A_5 = -2 n_{qq}2 n_{qq}3 - 2 n_{qq}1 n_{qq}4 + 2 I_s^2 d_d 1 d_d 4 + 2 I_s^2 d_d 2 d_d 3$$
$$A_4 = -n_{dd}2^2 - 2 n_{qq}2 n_{qq}4 + 2 I_s^2 d_d 1 d_d 5 + 2 I_s^2 d_d 2 d_d 4$$
$$- 2 n_{dd}1 n_{dd}3 - 2 n_{qq}1 n_{qq}5 + I_s^2 d_d 3^2 - n_{qq}3^2$$
$$A_3 = -2 n_{qq}3 n_{qq}4 + 2 I_s^2 d_d 2 d_d 5 - 2 n_{qq}2 n_{qq}5 + 2 I_s^2 d_d 3 d_d 4$$
$$A_2 = -2 n_{qq}3 n_{qq}5 - 2 n_{dd}2 n_{dd}3 + I_s^2 d_d 4^2 + 2 I_s^2 d_d 3 d_d 5 - n_{qq}4^2$$
$$A_1 = -2 n_{qq}4 n_{qq}5 + 2 I_s^2 d_d 4 d_d 5$$
$$A_o = -n_{qq}5^2 + I_s^2 d_d 5^2 - n_{dd}3^2 \quad .$$

Equation (3.14) allows the system to be solved in closed form without using iterative techniques. The procedure involves first specifying a particular operating frequency, as well as the value of the capacitor and load inductance. Next, the stator current $I_s$ is specified. With the current known, the IPM machine parameters $L_q$ , $L_d$ , and $\lambda_e$ can be found using Equation (2.32). The roots of Equation (3.14) are then determined. The root which is positive and real is the appropriate value of $R_L$. The solution for the d and q axis currents can then be found using Equation (3.12). Next, the d and q axis voltages of the IPM can be found using Equation (3.9) or Equation (3.10). Either one should give the same solution and, therefore, comparing the two offers a useful check as to the accuracy of the calculation. The load voltages and currents can then be found using Equations (3.8), (3.3), and (3.5). The entire procedure can be repeated for stator current values ranging from near zero amperes up to a relatively high current.

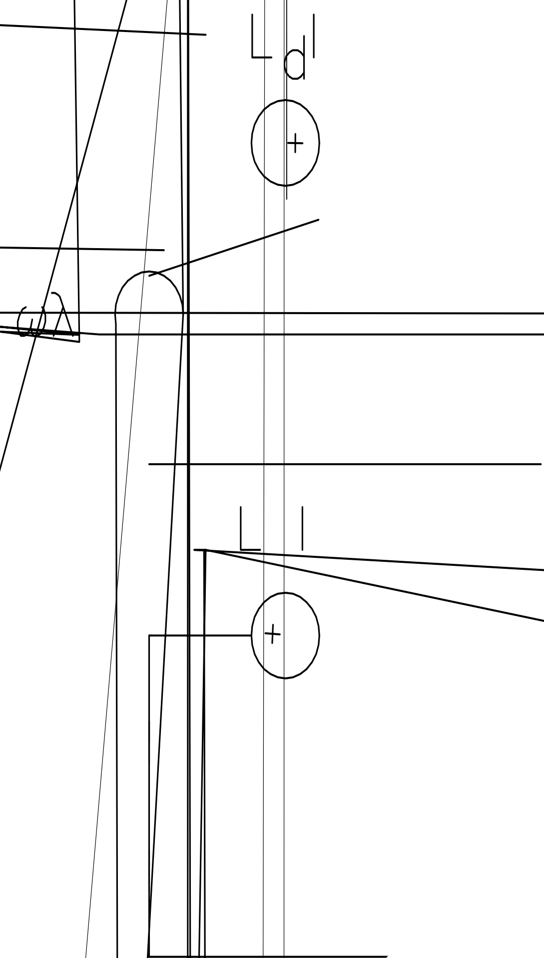


Figure 3.3. Equivale

resistive load wi

The d-q current equations can now be substituted ( as was done for the case of the RL load) into the current equation

$$I_s^2 - I_{qs}^2 - I_{ds}^2 = 0 \ . \tag{3.17}$$

After substitution, and solving the equation in terms of the load resistance gives the fourth order equation

$$B_0 + \sum_{n=1}^{4} ( B_n R_L^n ) = 0 \tag{3.18}$$

where

$$
\begin{aligned}
B_0 &= r_d 1^2 - r_q 1^2 + I_s^2 d_r 1^2 \\
B_1 &= 2 I_s^2 d_r 2 d_r 1 - 2 r_q 2 r_q 1 \\
B_2 &= - 2 r_d 2 r_d 1 + 2 I_s^2 d_r 3 d_r 1 + I_s^2 d_r 2^2 - 2 r_q 3 r_q 1 - r_q 2^2 \\
B_3 &= 2 I_s^2 d_r 3 d_r 2 - 2 r_q 3 r_q 2 \\
B_4 &= - r_d 2^2 + I_s^2 d_r 3^2 - r_q 3^2
\end{aligned}
$$

With the operating frequency and value of capacitance given, the load resistance can be solved for a particular stator current value.

## 3.4 Mathematical Model of IPM Feeding an R Load

The equivalent circuit of the PM machine feeding a purely resistive load is shown in Figure 3.4. The derivation for this system is quite easily accomplished by setting the capacitance equal to zero. With this done,

$$I_{qs} = \frac{s_q 2\, R_L + s_q 1}{e_r 3\, R_L^2 + e_r 2\, R_L + e_r 1}$$
$$I_{ds} = \frac{s_d 1}{e_r 3\, R_L^2 + e_r 2\, R_L + e_r 1} \quad , \tag{3.19}$$

where

$$s_q 1 = -\,\omega\, \lambda_e\, r_s\ , \quad s_q 2 = -\,\omega\, \lambda_e\ ,$$
$$e_r 1 = r_s^2 + \omega\, L_d + \omega\, L_q\ , \quad e_r 2 = 2\, r_s\ , e_r 3 = 1$$
$$s_d 1 = -\,\omega^2\, \lambda_e\, L_q \quad .$$

The d and q axis currents can now be combined using the property

$$I_s^2 - I_{qs}^2 - I_{ds}^2 = 0 \quad , \tag{3.20}$$

which, after simplification gives the fourth order equation

$$C_0 + \sum_{n=1}^{4} (C_n\, R_L^n\,) = 0 \quad , \tag{3.21}$$

where

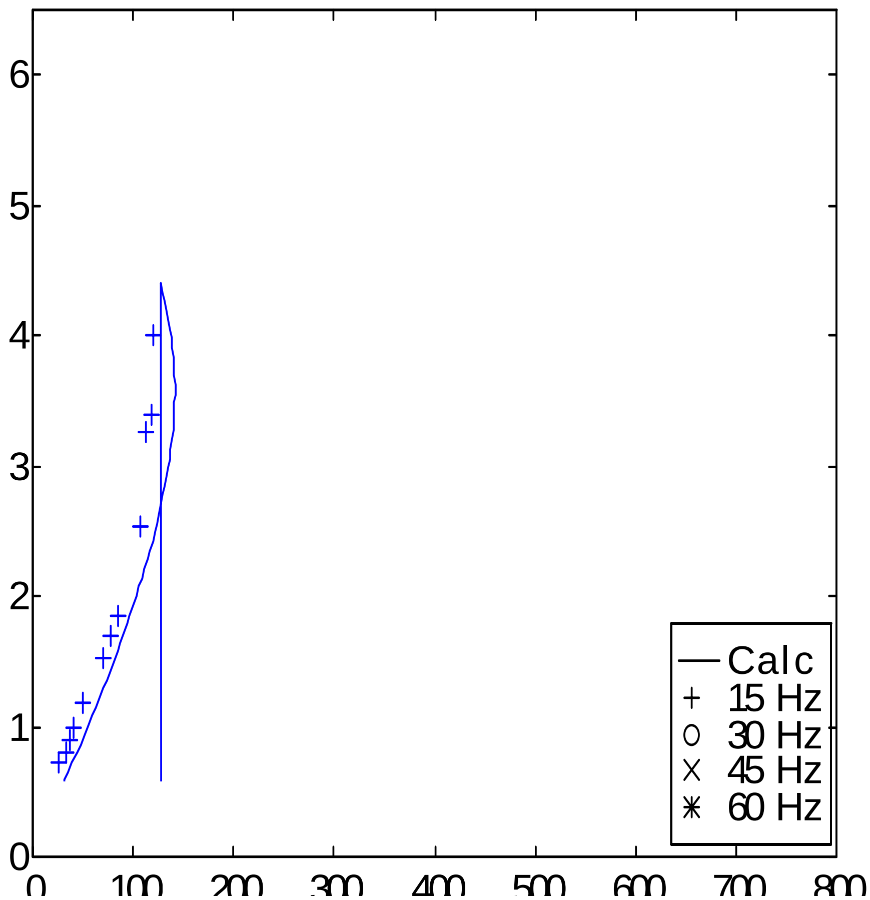
$$C_0 = s_d 1^2 - s_q 1^2 + I_s^2 e_r 1^2$$
$$C_1 = 2 I_s^2 e_r 2 e_r 1 - 2 s_q 2 s_q 1$$
$$C_2 = - 2 s_d 2 s_d 1 + 2 I_s^2 e_r 3 e_r 1 + I_s^2 e_r 2^2 - s_q 2^2$$
$$C_3 = 2 I_s^2 e_r 3 e_r 2$$
$$C_4 = I_s^2 e_r 3^2 \ .$$

It is interesting to note that Equation (3.21) can actually be reduced to a second order term by noticing that, since the load and stator resistances are in series, the load resistance and the stator resistance can be combined to form a resistance called $R_{LT}$. If the load resistance $R_L$ is replaced by the term $R_{LT}$ and, since the stator resistance is contained in the term $R_{LT}$ , $r_s$ is set equal to zero, then the terms $C_1$ and $C_3$ become equal to zero . The solution can then be written as

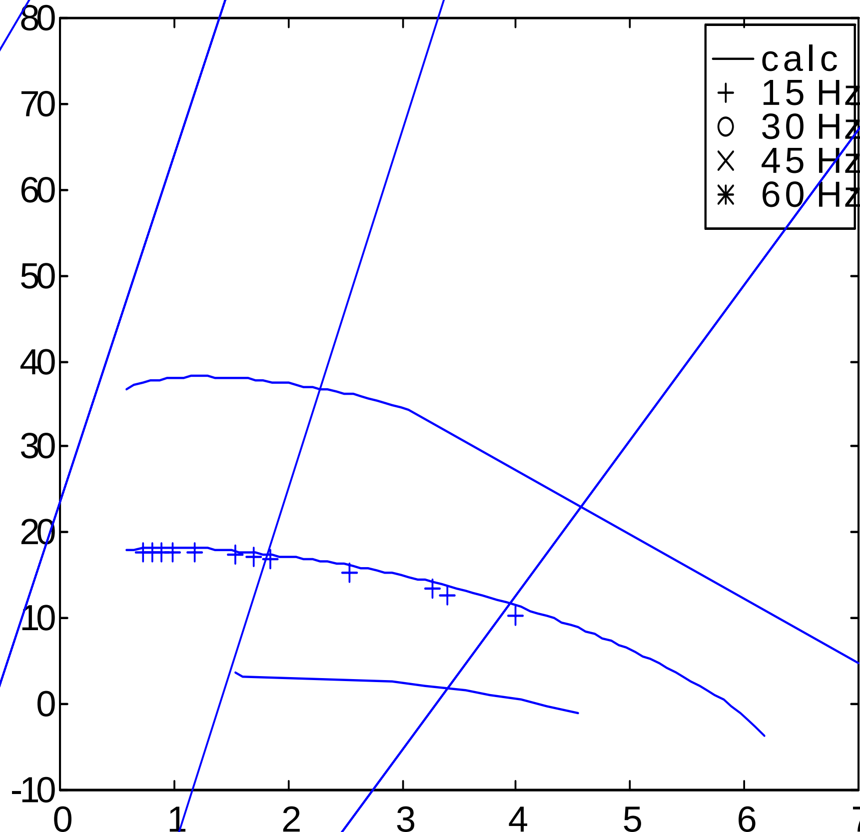
and the solution $R^2_{LT}$ can be solved as a simple quadratic equation. Taking the square root of this solution gives $R_{LT}$, and, with $r_s$ already known, $R_L$ can be found by subtracting the stator resistance from $R_{LT}$.

Figure 3.5. Measured and calculated line to neutral generator voltage vs load power output for various generator frequencies

Figure 3.6 shows the experimental and calculated results of the line current vs the output power. The lower part of the curves ( where the current is low ) is where one would ideally like to operate the machine. It is interesting to note that the maximum power point occurs around 4.1 amperes regardless of the operating frequency. This knowledge would be useful to have if a control or protection device scheme were implemented with the system.



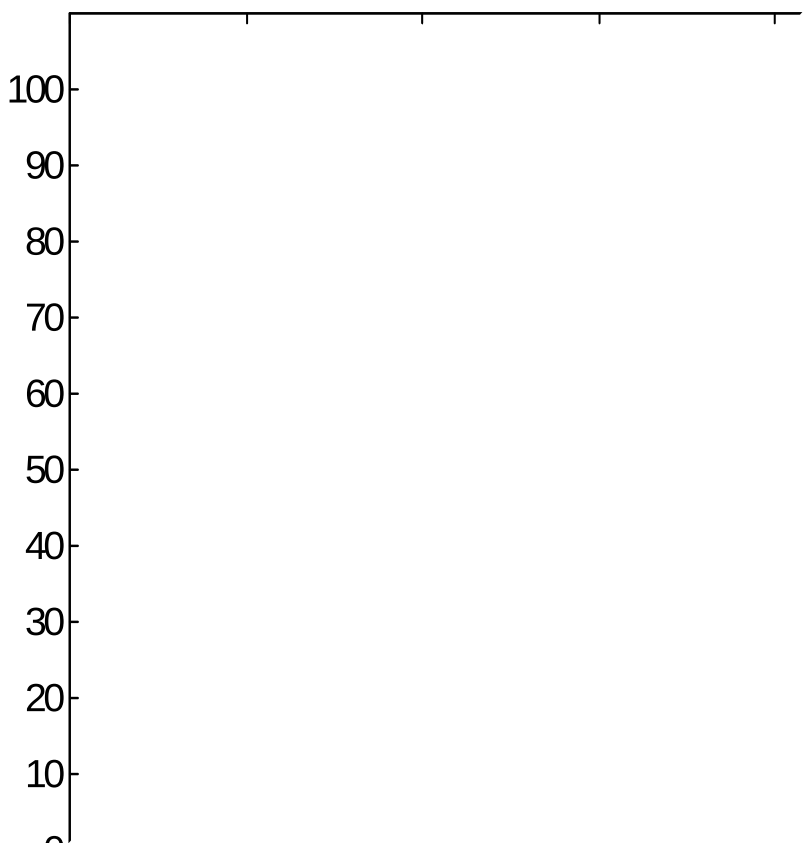
The plot of Figure 3.8 shows how the output pow



calc
15 Hz
30 Hz
45 Hz
60 Hz
80
70
60
50
40
30
20
10
0
-10
0
1
2
3
4
5
6
7

### 3.5.2 IPM Feeding an R Load With Shunt Capacitor Compensation

The graph shown in Figure 3.10 plots the line to neutral voltage of the IPM vs the output power of the generator when a 30 μF in Y connection is connected at the terminals of the machine. Although the trend is the same as the plots shown in Figure 3.5, it can be that the voltage at 60 Hz for the capacitor case is almost 30 volts higher than wh

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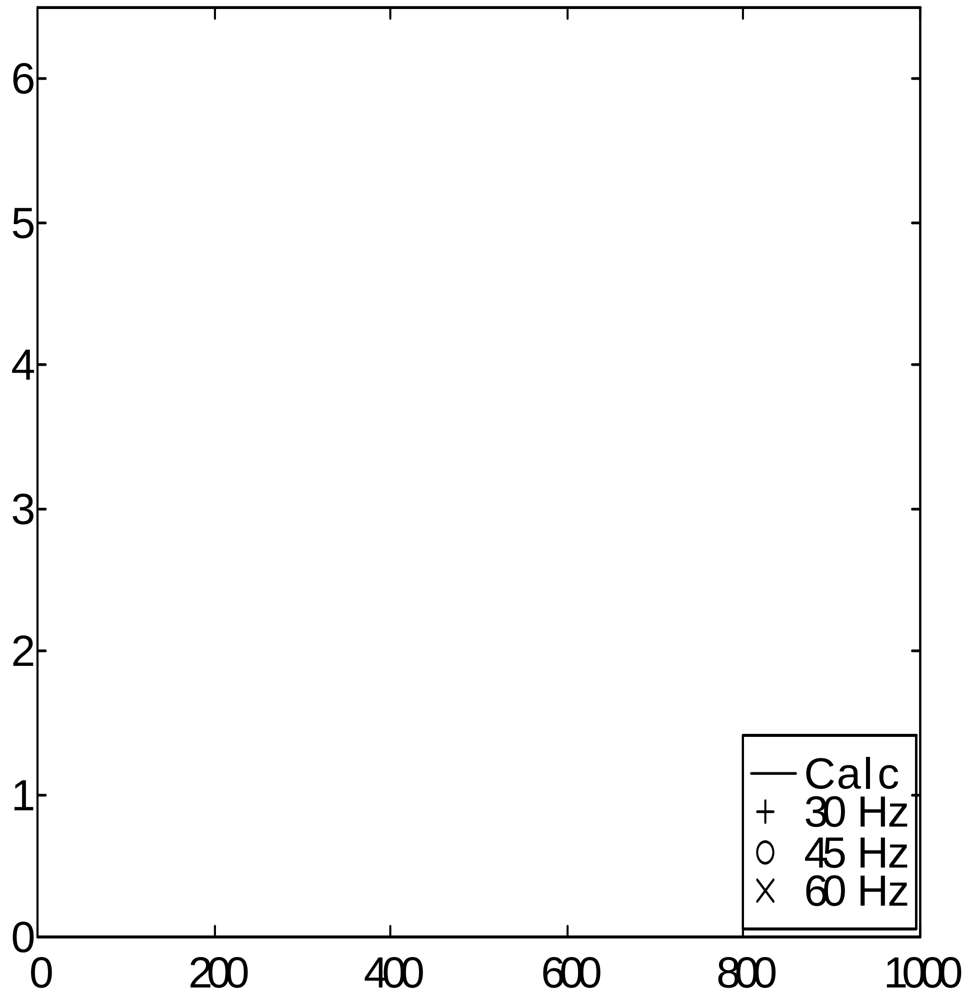
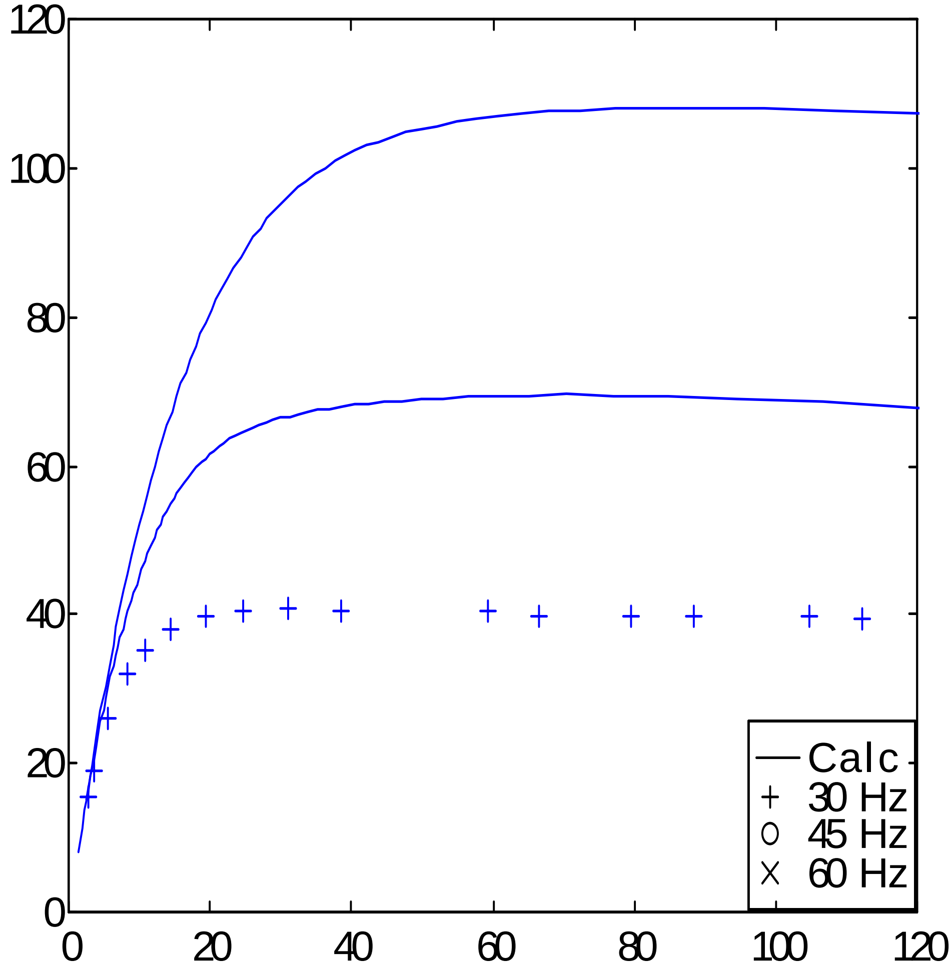
capacitor is used. In addition, there is an increase in power output of the machine of approximately 200 watts for the 60 Hz case. The increase in power and voltage is due to the capacitors providing reactive power to the IPM.



The plots shown in Figure 3.11 show the current of the IPM vs the output power. As was the case when no shunt capacitors were used ( see Figure 3.6) it can be seen that the maximum power point occurs when the line current is approximately 4.1 amperes regardless of the operating frequency.



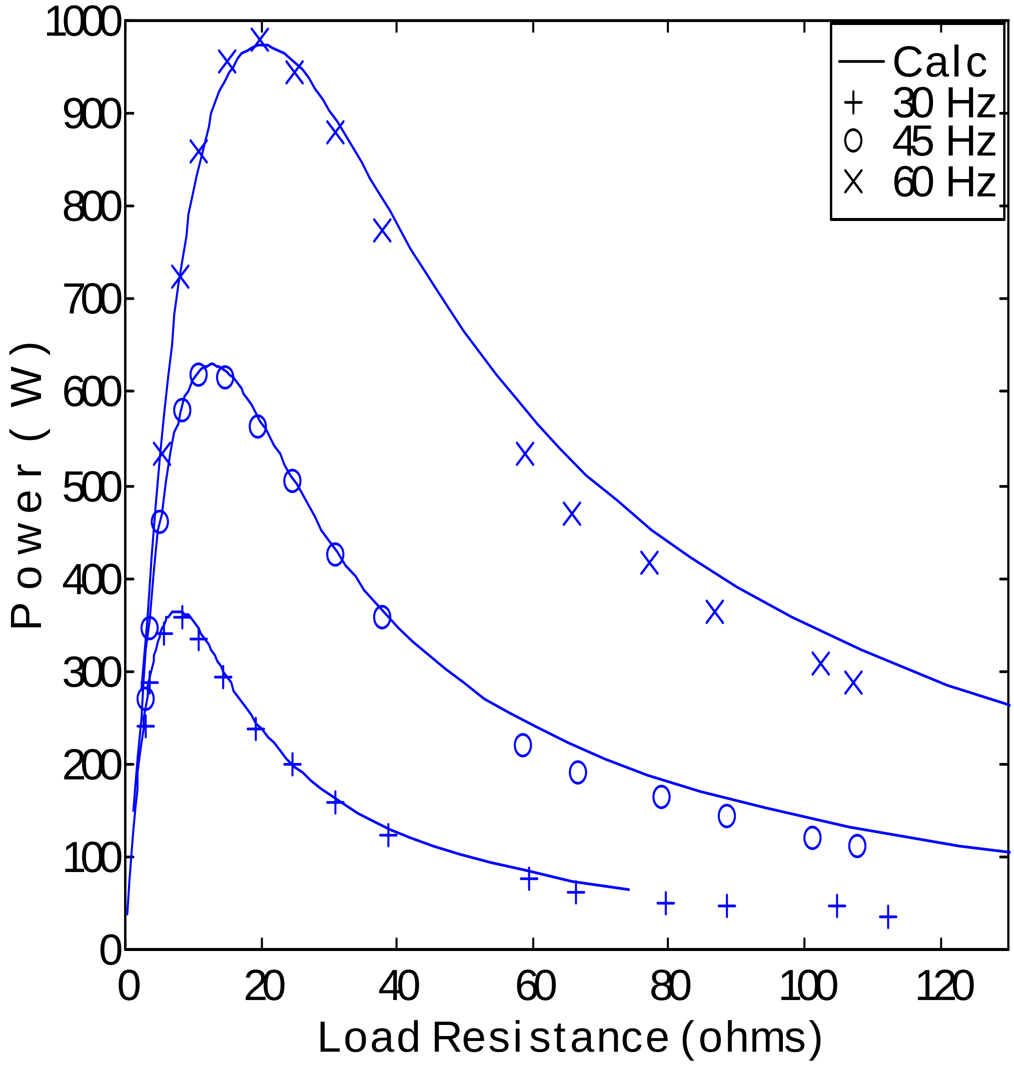



Figure 3.13. Measured and calculated output power vs load resistance for capacitor compensation scheme

Figures 3.12 and 3.13 display how the line to neutral voltage and the power into the load vary as the load resistance changes. The trends are exactly the same as those shown in Figures 3.7 and 3.8 (as one would expect) except for the fact that the magnitudes of both the line voltage and the output power have increased.

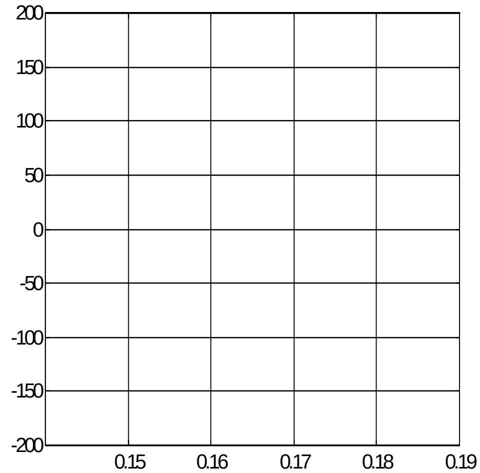
## 3.6 Comparison of Measured and Simulated Waveforms of IPM Machine with Shunt Capacitive Compensation Feeding a Resistive Load

This section includes the comparison between simulation and measured waveforms for the IPM generator feeding a resistive load. Shunt capacitors are used at the terminals of the generator to increase the power output and terminal voltage. Two cases will be looked at. The first is when the resistive load is such that high power is produced at the load, and the second is when the resistive load is very high and little power flows in the load.

### 3.6.1 IPM Generator Feeding a Small Resistive Load

This section looks at measured and simulated waveforms when the IPM is operating at near its maximum power output point. All of the waveforms in this and the next section are at the frequency of a 45 Hz. Since the IPM is a 4 pole machine, then this means that the shaft speed is 1350 rpm. The load resistance for the waveforms presented in this section is 22 Ω.

Figure 3.15 shows the simulated and measured line to line voltage waveforms for the 22 Ω load. The waveforms are fairly close with the peak voltage of the measured value being only slightly higher than the peak of the simulated waveform.




I. Measurement II. Simulation

Figure 3.15. Measured and simulated line to line voltage waveforms for the generator feeding a 22 Ω resistive load. Rotor speed=1350 rpm. Measured waveform scale: voltage: 50v/div, time 5ms/div

Figure 3.16 shows the measured and simulated waveforms of the generator current. Like the measured voltage waveform, it can be seen that the generator current waveform is not perfectly sinusoidal (whereas the simulated waveforms of both the current and voltage

inusoidal) . Nevertheless, the magnitudes of the measured and simulated generator

e reasonably close for this operating condition.

asured and simulated waveforms of the current flowing into the load are

7. Again, the measured waveform shows a slight distortion, but the

vaveforms are reasonably close.

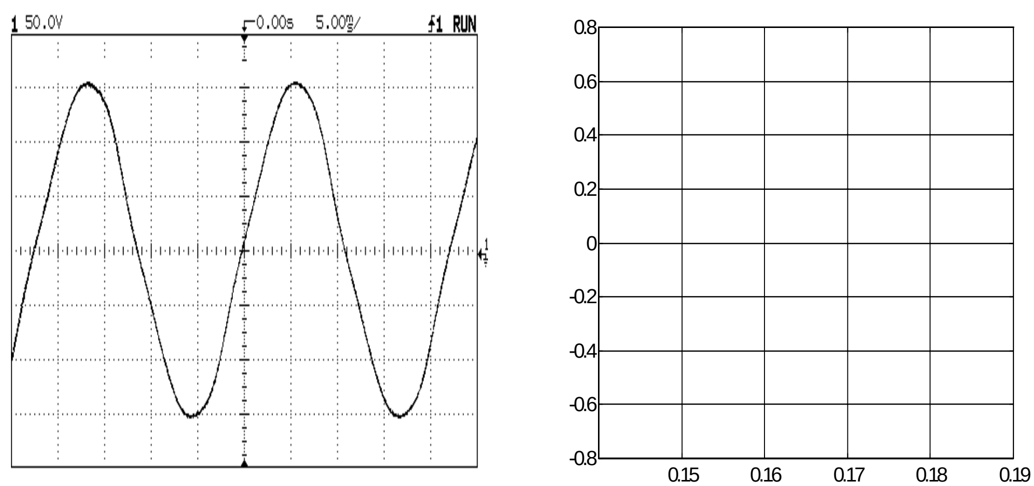
I. Measurement

II. Simulation

Figure 3.16. Measured and simulated ge

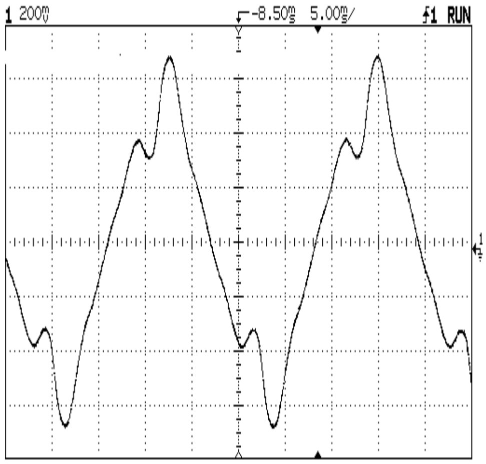
### 3.6.2 IPM Generator Feeding a Large Resistive Load

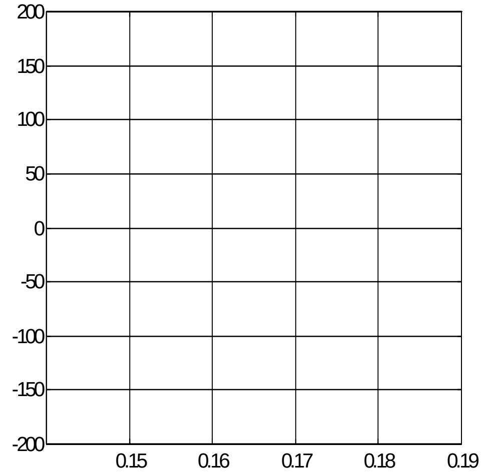
In this section, measured and simulated waveforms for the condition of a 205 Ω per phase load are described. The operating frequency is 45 Hz and the shunt capacitors are Y-connected 30 µF. Figures 3.18-3.20 display the measured and simulated waveforms of the line to line voltage, the generator current, and the load current. It is interesting to note that, while the measured line to line voltage of Figure 3.18 appears to be slightly less distorted than the voltage waveform for the heavy load condition ( see Figure 3.15), the generator current waveform of Figure 3.19 has become very distorted. It can be seen in Figure 3.20 that the load current is much less distorted than the generator current and, thus, it can be inferred from this that the shunt capacitors are serving as a filter.




I. Measurement II. Simulation

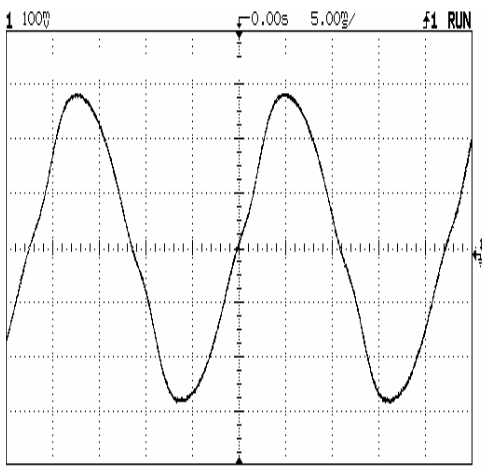
Figure 3.18. Measured and simulated waveforms of the line to line voltage for a 205Ω load. Rotor speed = 1350 rpm. Measured waveform scale: voltage: 50V/div, time 5ms/div






I. Measurement II. Simulation

Figure 3.19. Measured and simulated waveforms of generator current for a 205Ω load. Rotor speed=1350 rpm. Measured waveform scale: current 200 mA/div, time 5ms/div



I. Measurement II. Simulation

Figure 3.20. Measured and simulated waveforms of load current for a 205Ω load. Rotor speed=1350 rpm. Measured waveform scale: current: 100 mA/div, time 5ms/div