CHAPTER4

MODEL OF THREE-PHASE INVERTER

4.1 Introduction

In this chapter the three-phase inverter and its functional operation are discussed. In order to realize the three-phase output from a circuit employing dc as the input voltage a three-phase inverter has to be used. The inverter is build of switching devices, thus the way in which the switching takes place in the inverter gives the required output. In this chapter the concept of switching function and the associated switching matrix is explained. Lastly the alternatives as to how the inverter topologies can be formed is presented.

4.2 Switching Functions

It is well-known that some switching devices exist between the source and the load, the number of which depends on the circuit or the type of the load. In any case the number of switching devices are limited by the complexity. Even the densest circuit has one switch between an input line and the output line. If a converter has 'n' inputs and 'm' outputs the number of switching devices needed for energy conversion is equal to '$m\Delta n$'. These '$m\Delta n$' switching devices in the circuit can be arranged according to their connections. The pattern suggests a matrix as shown in Figure 4.1.

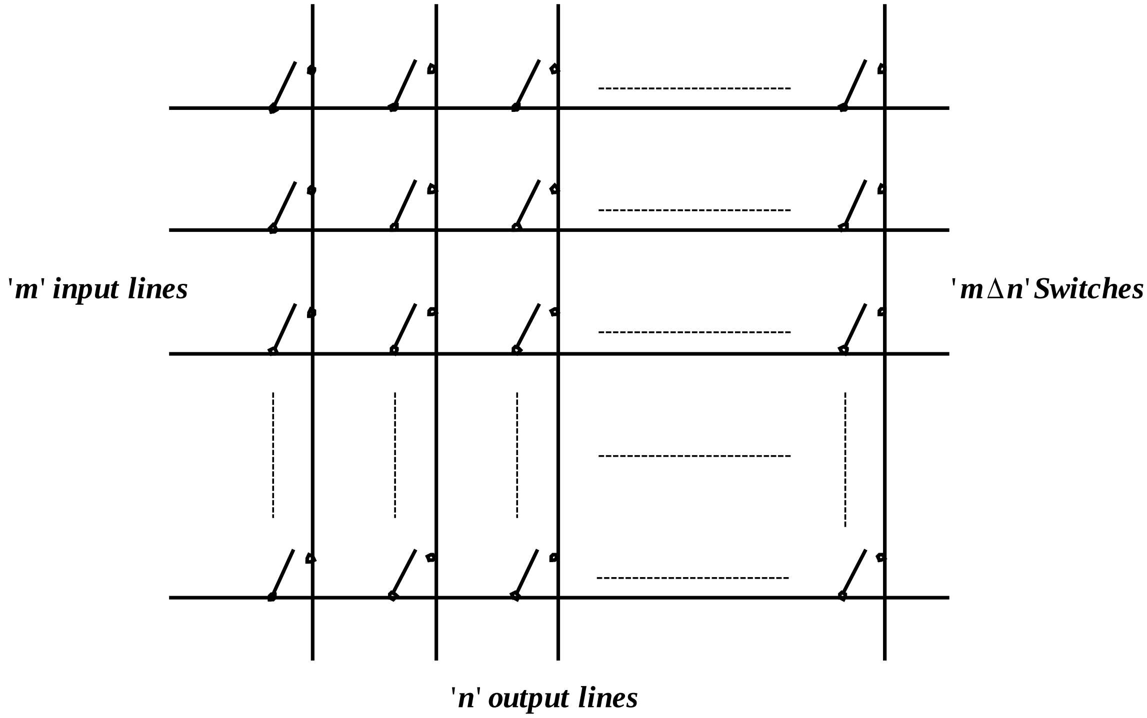


Figure 4.1: The general switch matrix

For example in realizing single phase to dc conversion the single phase has two terminals and the dc has two terminals thus a total of ‘2Δ2 = 4’ switches are required.

Most power electronic circuits are classified into two types [15]:

1. Direct switch matrix circuits: In these circuits any energy storage elements are connected to the matrix only at the input and output terminals. The storage elements effectively become a part of the source or the load. A full wave rectifier with an output filter is an example of a direct switch matrix circuit.
2. Indirect switch matrix converters also termed as embedded converters: In these circuits, the energy storage elements are connected within the matrix structure. There are usually very few energy storage elements in such case and the indirect switch

matrix circuits are often analyzed as cascade of two direct switch matrix circuits with storage elements in between.

A switch matrix provides a clear way to organize devices for a given application. It also helps to focus the effort in to three major problems areas. Each of these areas must be addressed effectively in order to produce a useful power electronic system.

1.The hardware problem $\Downarrow$ To build a switch matrix.

2. The software problem $\Downarrow$ Operate the matrix to achieve the desired conversion.

3.The interface problem $\Downarrow$ Add energy storage elements to provide the filters or intermediate storage necessary to meet the application requirements.

These problems can more effectively be understood by considering an example of converting ac to dc, in which the hardware problem is as to how many switches have to be used which depends on whether we are performing single phase to dc conversion or three phase to dc conversion.

The software problem lies in the fact that while operating these switches we need to obey the fundamentals laws of energy conversion like the Kirchhoff's voltage and the current laws and take care that these are not violated. Finally as this energy conversion process is not ideal we may not get exact dc voltage at the output terminals, which require the addition of some elements before the final dc voltage is obtained.

4.2.1 Reality Of Kirchhoff's Voltage Law

In determining how the switches operate in the switch matrix, care should be taken to avoid any danger in the operation of the circuit. Consider the circuit in Figure 4.2. This circuit can be operated with the switch 's' closed owing to the fact that when the switch is closed the sum of the voltages around the loop is not

4.2.2 Reality Of Kirchhoff's Current Law

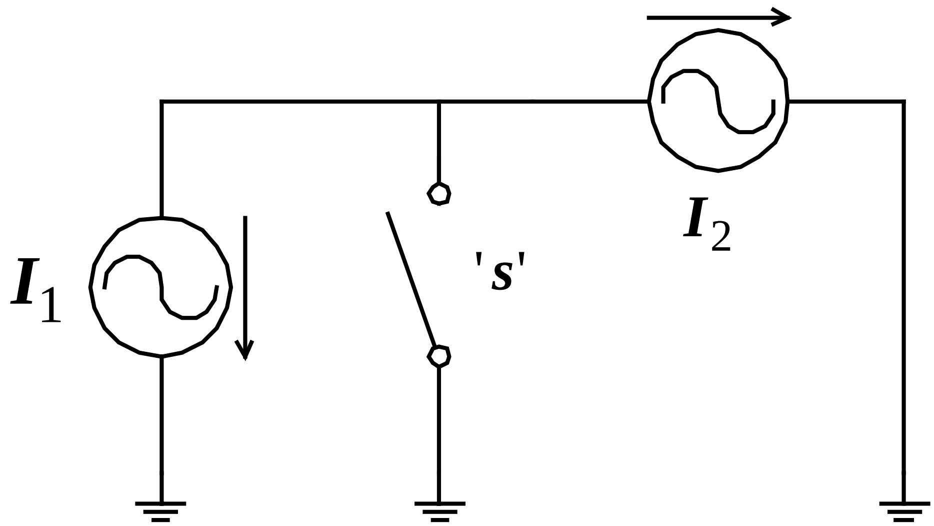


Figure 4.3: Demonstration of Kirchhoff's Current Law.

As is with voltages proper care is to be taken with currents too. According to Kirchhoff's Current Law (KCL) the sum of the currents at a node should be equal to zero at all tim

$\varpi T \mid 2\phi$

The average value of the term, 1/A can be brought within the summation to give an alternate form of this expansion,

$$H(\omega t) = \frac{1}{\pi}\sum_{n=-\infty}^{n=\infty}[\sin(n\pi/A)/n]\cos(n\omega t) \tag{4.6}$$

Equation 4.5 can be written equivalently as

$$H(\omega t) = \frac{1}{A} + \frac{2}{\pi}\sum_{n=1}^{n=\infty}[\sin(n\pi/A)/n]\cos(n(\omega t - 2k\pi/A)) \quad . \tag{4.7}$$

The above equation is true owing to the fact that $\cos(\omega t) = \cos(2\pi - \omega t)$. In the above expressions A can be an integer, rational or an irrational number. When A is an integer multiple of A then the term $\sin(n\pi/A)$ vanishes. In Equation 4.7 all those terms in which n is an integer multiple of A vanish, and so it reduces to a fundamental component and a time varying term. Thus the switching pulses can be represented as a dc component and a cos or sine varying term as in Equation 4.8,

$$S = \frac{1}{2}(1 + M) \tag{4.8}$$

where M is called the modulation signal, which can be any sine or cos term (in accordance to Fourier series) depending on the control we want to implement. The more general fundamental component for M is given as

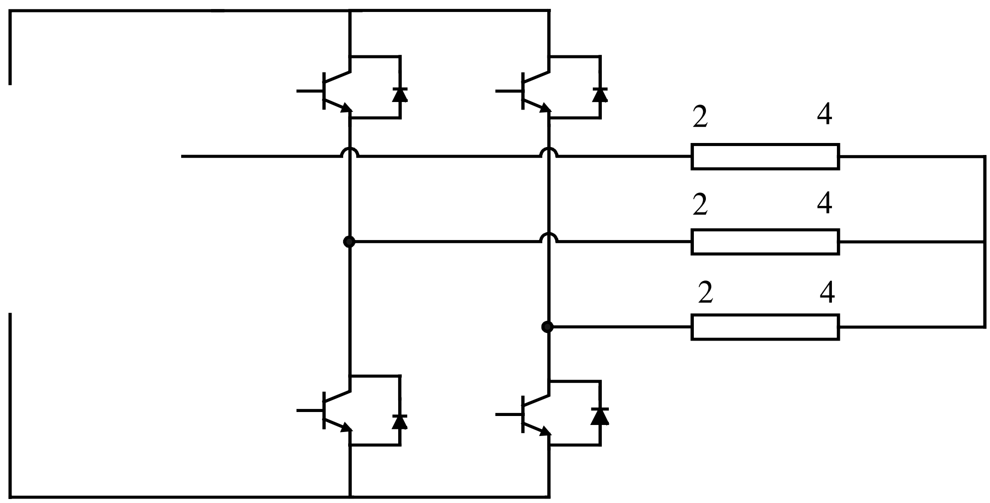
$$M = m_a\cos(\omega t - \zeta) \tag{4.9}$$

in the above expression m_a is called the modulation index which can vary from 0 to 1 and for values of m_a less than '1' a linear modulation range is supposed to exist and for values of m_a greater than '1' it is operated in the over modulation range.

4.3 Three-Phase Inverter

The dc to ac converters more commonly known as inverters, depending on the type of the supply source and the related topology of the power circuit, are classified as voltage source inverters (VSIs) and current source inverters (CSIs). The single-phase inverters and the switching patterns were discussed elaborately in Chapter two and so the three phase inverters are explained in detail here.

Three-phase counterparts of the single-phase half and full bridge voltage source inverters are shown in Figures 4.4 and 4.5. Single-phase VSIs cover low-range power applications and three-phase VSIs cover medium to high power applications. The main purpose of these topologies is to provide a three-phase voltage source,


2
4
2
4
2
4

$$S_{21}\ 2\ S_{22}\ |1 \tag{4.11}$$

$$S_{31}\ 2\ S_{32}\ |1 \tag{4.12}$$

Table 4.1: The switching states in a three-phase inverter.

S_{11}	S_{12}	S_{31}	V_{ab}	V_{bc}	V_{ca}
0	0	0	0	0	0
0	0				

[redacted]

V

4.3.1.Sinusoidal PWM in Three-Phase Voltage Source Inverters

As in the single phase voltage source inverters PWM technique can be used in three-phase inverters, in which three sine waves phase shifted by 120∀ with the frequency of the desired output voltage is compared with a very high frequency carrier triangle, the two signals are mixed in a comparator whose output is high when the sine wave is greater than the triangle and the comparator output is low when the sine wave or typically called the modulation signal is smaller than the triangle. This phenomenon is shown in Figure 4.6. As is explained the output voltage from the inverter is not smooth but is a discrete waveform and so it is m

The modulation signals are thus selected so meet some specifications, like harmonic elimination, higher fundamental component and so on. The phase voltages can be obtained from the line voltages as,

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} V_{an} - V_{bn} \\ V_{bn} - V_{cn} \\ V_{cn} - V_{an} \end{bmatrix} \tag{4.24}$$

which can be written as a function of the phase-voltage vector $[V_{an}\ V_{bn}\ V_{cn}]^T$ as

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \tag{4.25}$$

Equation 4.25 represents a linear system where the unknown quantity is the vector $[V_{an}\ V_{bn}\ V_{cn}]^T$, but the matrix is singular and so the phase voltages cannot be found from matrix inversion. However since the phase voltages add to zero, the phase load voltages can be written as

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}. \tag{4.26}$$

which implies

$$V_{bn} \mid \frac{1}{3}(V_{bc}\ 4V_{ab}) \tag{4.29}$$

$$V_{cn} \mid \frac{4\,1}{3}(2V_{bc}\ 2\,V_{ab})\,. \tag{4.30}$$

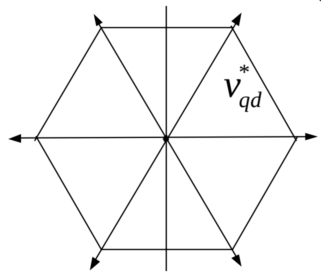
4.3.2 Generalized Discontinuous PWM

In the Equations from 4.16 to 4.18 V_{an}, V_{bn} , and V_{cn} are the phase voltages of the load while the voltage of the load neutral to the inverter reference is V_{no}. An alternative carrier based discontinuous modulation scheme is obtained by using the Space Vector methodology to determine the expression for V_{no} [17].

Table 4.2. The eight switching possibilities for the voltage source inverter along with the stationary reference frame voltages

S_{11}	S_{12}	S_{31}	V_{qs}	V_{ds}	V_{os}
0	0	0	0	0	$-V_{DC}/2$
0	0	1	$-V_{DC}/\cdot 3$	$V_{DC}/\cdot 3$	$-V_{DC}/6$
0	1	0	$-V_{DC}/3$	$-V_{DC}/\cdot 3$	$-V_{DC}/6$
0	1	1	$-2V_{DC}/3$	0	$V_{DC}/6$
1	0	0	$2V_{DC}/3$	0	$-V_{DC}/6$

The eight possible switching states for the voltage source inverter along with the qd transformation in stationary reference frame are shown in Table 4.2. The voltages as given in Table 4.2 can be graphically represented as a hexagon whose sides are the switching states.



times the two active switching modes are utilized is less than the switching period; in which case the remaining time is occupied by using the two null vectors, U_0 and U_7

Table 4.3: Expressions for the neutral voltage for the six sectors

Sector	**Neutral Voltage**


S11
1
0
(a)
S21
1
0
(b)
S31
1
0
(c)

The square wave phase voltages with respect to the fictitious dc center tap can be expressed using Fourier series as,

$$V_{ao} | \frac{2V_{DC}}{\phi}[\cos \varpi t \, 4 \, \frac{1}{3}\cos 3\varpi t \, 2 \, \frac{1}{5}\cos 5\varpi t \, 4 ....] \quad (4.36)$$

$$V_{bo} | \frac{2V_{DC}}{\phi}[\cos(\varpi t \, 4 \, \frac{2\phi}{3}) \, 4 \, \frac{1}{3}\cos 3(\varpi t \, 4 \, \frac{2\phi}{3}) \, 2 \, \frac{1}{5}\cos 5(\varpi t \, 4 \, \frac{2\phi}{3}) \, 4 ....] \quad (4.37)$$

$$V_{co} | \frac{2V_{DC}}{\phi}[\cos(\varpi t \, 2 \, \frac{2\phi}{3}) \, 4 \, \frac{1}{3}\cos 3(\varpi t \, 2 \, \frac{2\phi}{3}) \, 2 \, \frac{1}{5}\cos 5(\varpi t \, 2 \, \frac{2\phi}{3}) \, 4 ....]. \quad (4.38)$$

The line voltages can thus be obtained from the phase voltages as

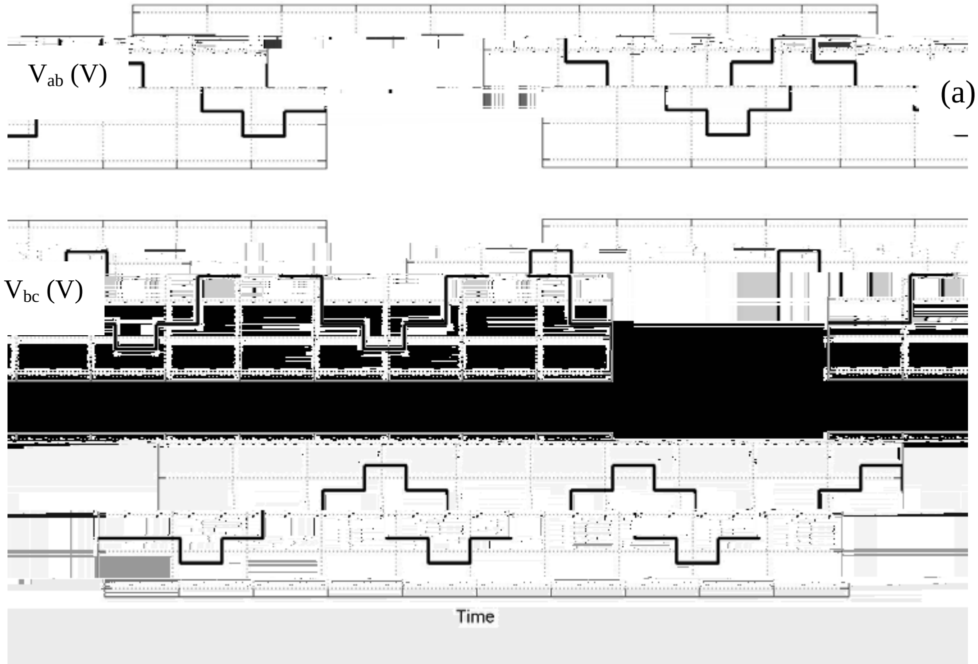
$$V_{ab} | V_{an} \, 4 \, V_{bn}$$

$$| \frac{2\sqrt{3}V_{DC}}{\phi}[\cos(\varpi t \, 2 \, \frac{\phi}{6}) \, 2 \, 0 \, 4 \, \frac{1}{5}\cos 5(\varpi t \, 2 \, \frac{\phi}{6}) \, 4 \, \frac{1}{7}\cos 7(\varpi t \, 2 \, \frac{\phi}{6}) \, 2 ...] \quad (4.39)$$

$$V_{bc} | V_{bn} \, 4 \, V_{c}$$

$$\sqrt{} \quad \cos 7(\quad \frac{}{2}) \quad ...]$$

7q s B



V_ab (V)
(a)
V_bc (V)
Time

The dc current from the source is as given in Equation 4.45

$$I_{DC} = I_a S_{11} + I_b S_{21} + I_c S_{31} \tag{4.45}$$

The switching functions S_{11}, S_{21} and S_{31} can be expressed in terms of modulation signals as

$$S_{11} = \frac{1}{2}(1 + M_{11}) \tag{4.46}$$

$$S_{21} = \frac{1}{2}(1 + M_{21}) \tag{4.47}$$