CONCISE MODULATION STRATEGIES FOR FOUR-LEG VOLTAGE SOURCE INVERTERS

ABSTRACT : The continuous and discontinuous pulse-width modulation (PWM) schemes and a novel Space Vector modulation methodology are proposed in this paper for four-leg DC-AC inverters. Using a Space Vector definition that includes the zero sequence voltage component and partitioning the feasible sixteen modes into two separate sets – one set having zero sequence voltages with positive magnitudes and the other set with the zero sequence voltages with negative magnitudes – the novel Space vector implementation technique is determined as also the discontinuous carrier based PWM scheme. For the continuous carrier based PWM scheme, the indeterminate defining output voltage equations expressed in terms of the existence functions of the switching devices are solved using an optimization technique. The modulation schemes determined are shown by experimental results to synthesis any desirable balanced or unbalanced three-phase voltage sets when operating in the linear modulation region.

I. INTRODUCTION

Stand-alone three-phase power supplies with high waveform quality and performance are increasingly required for critical applications such as military and medical equipment, satellite earth station, large scale computer systems, distributed power systems, and for rural electrification schemes in remote locations. In view of the possible imbalances in the loads which are becoming nonlinear, four-leg DC/AC inverters are recommended, especially in applications where the neutrals of the loads are accessible. In certain applications, front-end three-phase diode rectifiers fed from a generator or alternative energy sources, such as solar systems, fuel-cells or battery banks provides the input DC source to the four-legged inverter. It is now standard procedure to ensure voltage, current regulation or power quality improvement (mitigation of harmonics, etc.) through either pulse-width modulation or space vector inverter control schemes. A space vector modulation scheme fashioned after the classical qdo stationary reference frame space vector methodology has been proposed and shown by computer simulations to be capable of balancing load voltages and improve current quality [1-3,6]. In view of the alleged inability of the well- known 3x3 abc-qdo stationary reference transformation to reflect the fourth degree of freedom that the four inverter legs provide in the modeling of the four-leg inverters, a rather complicated 4x4 or quad stationary reference frame transformation has been proposed for inverter modeling which, with synchronous reference frame controllers, is used to experimentally showcase the possibility of voltage regulation under nonlinear or unbalanced three-phase load conditions [4-5].

The paper contributes to the development of both the space vector and carrier-based modulation schemes for the four-leg DC/AC inverters. The definition of the problem permits the use of the classical qdo transformations; however, unlike the classical space vector where the zero sequence voltages are ignored, they are used here for calculating the turn-on times of the devices. With the expressions for the times the devices are turned on for a desired voltage set, the expressions for the discontinuous modulation signals for the devices are determined. Various discontinuous modulation schemes for three-phase inverters have been investigated [7-10]. Furthermore, using the inverter voltage equations expressed in terms of the existence functions of the devices and an optimization methodology based on Moore-Penrose inverse, the expressions for the modulating signals constituting the carrier-based continuous PWM for all the eight switching devices are explicitly determined. The methodologies proposed for determining the carrier-based and space vector modulation set forth are considered to be novel and are extendable for the determination of modulation schemes for other current or voltage source converters including the multi-level and converters with reduced component counts (minimalist converters).

II. CONTINUOUS PWM MODULATION

Fig. 1 shows the circuit topology of the four-leg voltage source DC/AC inverter in which the fourth leg, in general is connected through an impedance to

In equations in (1), V_{an},$V_{bn,}$$V_{cn}$ are the desired phase voltages of the load and the phase voltage of the neutral impedance connected to the fourth leg is V_{dn}. The voltage between a reference 'o' of the inverter and the neutral of the load is denoted by $V_{no.}$ In order to prevent short-circuiting the DC source and thereby not violate the Kichoff's voltage law, $\mathbf{S_{ip}}$ and $\mathbf{S_{in}}$ cannot be turned on at the same time. Hence, Kirchoff's law constraints the existence function such that $\mathbf{S_{ip}} + \mathbf{S_{in}}$

$M_{ap} = 1/2\ (3V_{ann} – V_{bnn} – V_{cnn}\ - V_{dnn})$, $V_{ann} = V_{an}/ V_d$

$M_{bp} = 1/2\ (-V_{ann} + 3V_{bnn} – V_{cnn} - V_{dnn})$, $V_{bnn} = V_{bn}/ V_d$

$M_{cp} = 1/2\ (-V_{ann} - V_{bnn} + 3V_{cnn} - V_{dnn})$, $V_{cnn} = V_{cn}/ V_d$

$M_{dp} = 1/2\ (-V_{ann} - V_{bnn} - V_{cnn} + 3V_{dnn})$, $V_{dnn} = V_{dn}/ V_d$ (3)

M_{ip} are the continuous PWM modulation signals for the top devices of the four inverter legs. These signals are compared with a high frequency triangle carrier waveform (ranging from + 1 to –1) to generate the PWM switching pulses for the base drives of the switching devices represented by the existence functions S_{ip} .

III. SPACE VECTOR PWM

Respecting the Kirchoff's voltage law, which implies that the top and bottom switching devices of an inverter leg cannot be turned on at the same time, there are 16 feasible switching modes of the four-leg inverter which are enumerated in Table I [2]. The stationary reference frame qdo voltages of the switching modes are expressed in the complex variable form as ($a = e^{j\beta}$, $\beta = 120°$)

$V_{qds} = 2/3(V_{an} + aV_{bn} + a^2V_{cn})$, $V_o = 1/3(V_{an} + V_{bn} + V_{cn})$ (4)

Using the phase to reference voltages V_{ao}, V_{bo}, V_{co} and V_{do} for each switching mode and equations in (2), the components of the stationary reference frame V_{qdo} given in (5) are also shown in Table I.

$V_{qs} = 1/6(\ 2S_{ap} – S_{bp} – S_{cp} - 2S_{an} + S_{bn} + S_{cn}\ )V_d$,

$V_{ds} = 1/2\sqrt{}$

It is evident from Table I that the 16 switching modes can be divided into three broad divisions. Modes ka and kb (k =1,2,3,4,5,6) have the same q and d axis voltages; however the values of the zero sequence voltages for modes ka are negative and those of kb are positive. Modes 7 and 8 are two null states while modes 9 and 10 are states with zero qd components having zero sequence voltages of opposite magnitude signs. We propose a space vector methodology based on the partitioning of modes a and b as shown in Fig. 2 where null states 7,8,9,10 are common to both. Since the inverters are used in systems with unbalanced and nonlinear loads, the zero sequence voltages for the switching modes must be included in the calculations and are therefore reflected in Fig. 2. In Fig. 2(a), the zero sequence voltages of the active modes are positive while they are negative in Fig. 2(b). In the sequel, Fig. 2(a) will be referred to as the positive (p) sequence space vector while Fig. 2(b) is the negative (n) space vector.

In classical space vector technique, a reference voltage V_{qd}^{*} located within the six sectors of the complex space vector in Fig. 2 is approximated instantaneously by time-averaging of six vectors comprising of two adjacent active switching modes and the two null modes 0,7 over the PWM sampling period T_s, which is much greater than the period of the reference signal. However, for the four-leg inverter, the reference voltage is approximated by time-averaging six switching modes comprising of the two active modes which are adjacent to the reference V_{qd}^{*}, and the four null voltage modes 7,8,9,10.

To realize a reference in sector I, switching 1,2,7,8,9,10 with voltages V_{qdo1}, V_{qdo2}, V_{qdo7}, V_{qdo8}, V_{qdo9}, V_{qdo10} are time-averaged while for another reference voltage in Sector V, switching modes 5,6,7,8,9,10 with voltages V_{qdo5}, V_{qdo6}, V_{qdo7}, V_{qdo8}, V_{qdo9}, V_{qdo10} are used. The normalized times the active modes (V_{qdoa} and V_{qdob}) are used are t_a and t_b respectively, t_d is the combined normalized time modes 9 and 10 are applied and the combined normalized time modes 7 and 8 are utilized is t_c. If mode 7 is applied for (1-κ)t_c , mode 8 for κt_c , mode 9 for γt_d and mode 10 for (1-γ) t_d , then $t_a + t_b + t_c + t_d = 1$, $0 \le \kappa \le 1$, $0 \le \gamma \le 1$.

$$V_{qdo}^{*} = V_{qdoa}\, t_a + V_{qdob} t_b + V_{qdo7}(1-\kappa) t_c + V_{qdo8}\kappa t_c + V_{qdo9}\gamma\, t_d + V_{qdo10}(1-\gamma)\, t_d \qquad (6)$$

Table I Switching modes and qdo voltages

S_{ap}	S_{bp}	S_{cp}	S_{dp}	$3V_q^*$

is turned on for the six sectors are determined and given in Table III. Note that the expressions for the modulation signals for the top devices in phases a,b,c

V. EXPERIMENTAL RESULTS

The proposed modulation schemes are practically implemented by means of a floating-point 26-MHz

Figure 4 : Experimental results : Generation of balanced three-phase voltages using continuous modulation scheme. V_{an} = 25 cos (377t), V_{bn} =25 cos (377t - 2π/3), V_{cn} = 25 cos (377t + 2π/3), V_d = 80V. (1-2) phase ‘a’ and ‘b’ voltages, (3-4) filtered phase ‘a’ and ‘b’ voltages, (5-8) phase modulating signals.

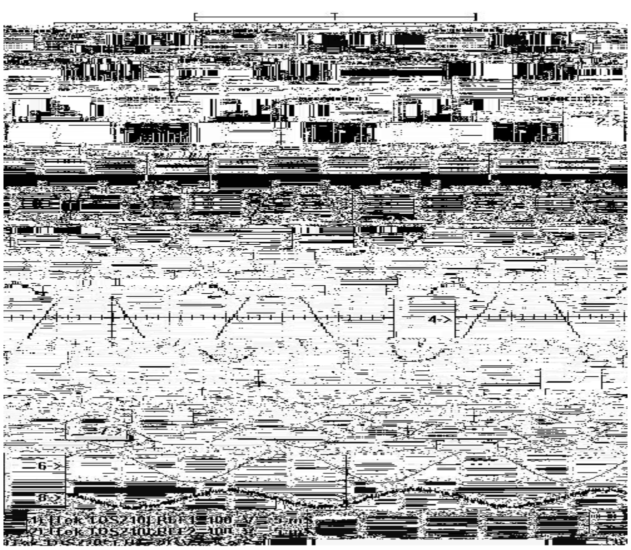

Figure 5: Experimental results : Generation of unbalanced three-phase voltages. Unbalanced voltages are V_{an} = 20 cos (377t), V_{bn} =25 cos (377t - 2π/3), V_{cn} = 25 cos (377t + 2π/3) V_d = 80V. (1-2) phase ‘a’ and ‘b’ voltages, (3-4) filtered phase ‘a’ and ‘b’ voltages, (5-8) phase modulating signals.

Figure 6 : Experimental results : Generation of balanced three-phase voltages using discontinuous modulation scheme. V_{an} = 25 cos (377t), V_{bn} =25 cos (377t - 2π/3), V_{cn} = 25 cos (377t + 2π/3) V_d = 80V. (1-2) phase ‘a’ and ‘b’ voltages, (3-4) filtered phase ‘a’ and ‘b’ voltages, (5-8) phase modulating signals.

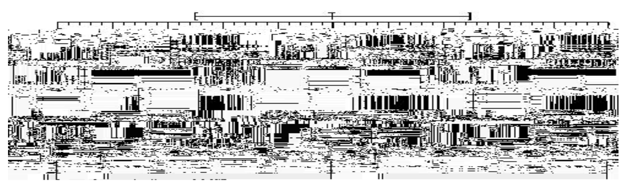

Figure 7 : Experimental results : Generation of unbalanced three-phase voltages using discontinuous modulation scheme. V_{an} = 25 cos (377t + π/12), V_{bn} =25 cos (377t - 2π/3), V_{cn} = 25 cos (377t + 2π/3) V_d = 80V. (1-2) phase ‘a’ and ‘b’ voltages, (3-4) filtered phase ‘a’ and ‘b’ voltages, (5-8) phase modulating signals.

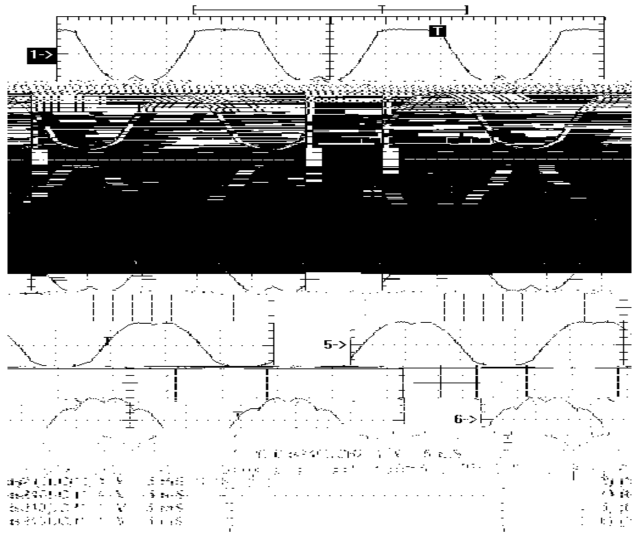

Figure 8: Generation of discontinuous modulation signals for phase ‘a’ top device. For balanced voltage set : (1) κ= 1, (2) κ= 0.5, (3) κ= 0 unbalanced voltage set : (4) κ= 1, (5) κ= 0.5,
(6) κ= 0

TABLE III : Modulation signals for the top devices

Sector	$\mathbf{M_{ap}}$	$\mathbf{M_{bp}}$	$\mathbf{M_{cp}}$	$\mathbf{M_{dp}(p)}$	$\mathbf{M_{dp}(n)}$	$\mathbf{t_d\,(p)}$	$\mathbf{t_d\,(n)}$
$0\geq \psi \geq 60$	$V_{abn}+\gamma \mathbf{t}_d + \kappa(1-V_{abn}-t_d)$	$\gamma t_d + \kappa(1-V_{abn}-t_d)$	$\kappa(1-V_{abn}-t_d)+V_{cbn}+\gamma \mathbf{t}_d$	$\kappa(1-V_{abn}-t_d)+(1-\gamma)\mathbf{t}_d$	$V_{abn}+\kappa(1-V_{abn}-t_d)+(1-\gamma)\mathbf{t}_d$	$V_{bdd}\chi$	$V_{add}\chi$
$60\geq \psi \geq 120$	$V_{abn}+\gamma t_d+\kappa(1-V_{cbn}-t_d)$	$\gamma t_d+\kappa(1-V_{cbn}-t_d)$	$V_{cbn}+\gamma t_d+\kappa(1-V_{cbn}-t_d)$	$\kappa(1-V_{cbn}-t_d)+(1-\gamma)\mathbf{t}_d$	$V_{cbn}+\kappa(1-V_{cbn}-t_d)+(1-\gamma)\mathbf{t}_d$	$V_{bdd}\chi$	$V_{cdd}\chi$

VI. CONCLUSIONS

Three modulation schemes – space vector and two carrier-based which are the continuous and discontinuous carrier-based modulation schemes – have been proposed for the four-leg DC/AC inverter for the generation of three-phase voltages which may be balanced or unbalanced. The space vector scheme proposed partitions the 16 modes of operation into two sets: the positive sequence space vector where the zero sequence voltage is positive while in the negative sequence space vector, the zero sequence voltages are negative. A reference three-phase voltage set expressed in terms of the three components of the qdo stationary reference frame is synthesized by time-averaging two active modes and four null states. This schemes permits the partitioning of the total times spent in the null modes which influences the performance of the modulator. The continuous modulation scheme on the other hand is derived using an optimization methodology while the averaging of the existence functions for each of the inverter sectors yields expressions for the discontinuous modulation signals. A variable in the expressions for the modulation signals in the discontinuous modulation scheme permits the partitioning of the timing of the zero sequence voltages achieved by defining the value of κ which must instantaneously lie between unity and zero.

Some confirmatory experimental results showing both the voltage waveforms and modulation signals have been provided to verify the modulation methodologies proposed. Unbalanced and balanced three-phase voltages were experimentally synthesized by both the new space vector and discontinuous modulation schemes.

The techniques set forth for determining the proposed modulation schemes have wider potential applications in the definition of modulation strategies of other converters. Our future publications will extend the above methodologies for operation in the over-modulation region, address the applicability of the variables κ and γ for improving the performance of the modulator, and the dynamic control of

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