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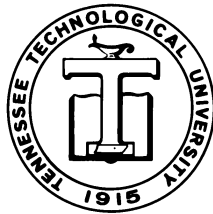
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# STEINHAUS PROPERTY IN BANACH LATTICES

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# STEINHAUS PROPERTY IN BANACH LATTICES

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Abstract.

## 3 Steinhaus property in Banach spaces

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Theorem 3.1. A Banach space  $(X, \|\cdot\|)$  has the Steinhaus property if and only if for every  $x, y \in S(X)$  and for every  $\epsilon > 0$  there exists  $z \in X$  such that  $\|z\| < \epsilon$  and  $\|x + z\|, \|y + z\| > 1 - \epsilon$ .

Proof. Let  $x, y \in S(X)$  and  $\epsilon > 0$ . Let  $A =$



is a  
 $\mathbb{R}$

$$\|x + z\| > \|x\| + \|z\|$$

X

#### 4 Steinhaus property in some Banach Lattices

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 $\mathbb{R}$

Lemma 4.1. A strictly monotone Banach lattice  $(E; \|\cdot\|_E)$  has the Steinhaus property if and only if for every  $x, y \in S(E)$ ,  $\langle x, y \rangle > 0$ ,  $x \not\leq y$  and for every  $\epsilon > 0$  there is  $z \in E$  with  $\|z\|_E = \epsilon$

