DEPARTMENT OF MATHEMATICS

TECHNICAL REPORT

RINGS OF INVARIANTS FOR SALINGAROS' VEE GROUPS

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Theorem 1 guarantees that there are finitely many invariants $f_1, \ldots, f_m$ such that $k[x_1, \ldots, x_n]^G = k[f_1, \ldots, f_m]$. Suppose that g_1 and g_2 are polynomials in $k[y_1, \ldots, y_m]$, then

$$g_1(f_1, \ldots, f_m) = g_2(f_1, \ldots, f_m) \iff h(f_1, \ldots, f_m) = 0\,; \tag{3}$$

where $h = g_1 - g_2$. It follows that uniqueness of the algebraic relations fails if and only if a nonzero polynomial $h \in k[y_1, \ldots, y_m]$ exists such that

$$h(f_1, \ldots, f_m) = 0\,. \tag{4}$$

Such nonzero polynomial h is a

Let $F = (x_1^2; x_2^2; x_1x_2)$ and let the new variables be $u; v; w$. Then the ideal of relations is obtained by eliminating $x_1; x_2$ from the equations

$$\begin{aligned} u &= x_1^2; \\ v &= x_2^2; \\ w &= x_1x_2: \end{aligned} \tag{10}$$

If we use the lex order with $x_1 > x_2 > u > v > w$, then a Groebner basis for the ideal

First consider $G_{3;0}$ or $G_{1;2}$. Then, using the degree 2 repres[illegible]gs of invariants $k[x_1; x_2]^{G_{3;0}}$ and $k[x_1; x_2]^{G_{1;2}}$ are computed. Each of them is [illegible]nomials. Later, we compute the corresponding syzygy ideals. Then, the sixt[illegible]se polynomials are exactly the sixty one polynomials in $y_1; \ldots; y_{13}$ generat[illegible]

$I_F = h y_1y_{12}y_{13} \quad y_2y_{10} \quad y_2y_{11} + y_4y_{11}; y_2^2 \quad y_1y_3 + y_3$ [illegible] $y_4 + y_2y_5 \quad y_4y_5; y_3^2 \quad y_1y_5 \quad 2y_5^2;$
$y_2y_4 \quad y_1y_5 \quad y_3y_5; y_3y_4 \quad y_2y_5 \quad y_4y_5; y_4^2 \quad y_3y_5$ [illegible] $+ y_3y_9 \quad 2y_5y_9; y_3y_6 \quad y_1y_8 + y_2y_9$
$y_4y_9; y_4y_6 \quad y_1y_9 \quad y_3y_9; y_5y_6 \quad y_2y_9; y_6^2 \quad y_1y_{10}$ [illegible] $y_2y_7 \quad y_1y_8 + y_2y_9 \quad y_4y_9; y_3y_7 \quad y_1y_9$
$2y_5y_9; y_4y_7 \quad y_2y_9 \quad y_4y_9; y_5y_7 \quad y_3y_9; y_6y_7$ [illegible] $2y_5y_{11}; y_2y_8 \quad y_1y_9 \quad y_3y_9; y_3y_8 \quad y_2y_9$ 1
$y_4y_9; y_4y_8 \quad y_3y_9 \quad 2y_5y_9; y_5y_8 \quad y_4y_9; y_6y_8$ [illegible] $y_7y_8 \quad y_2y_{11} \quad y_4y_{11}; y_8^2 \quad y_3y_{11} \quad 2y_5y_{11};$ 0874 T
$y_6y_9 \quad y_2y_{11}; y_7y_9 \quad y_3y_{11}; y_8y_9 \quad y_4y_{11}; y_9^2$ [illegible] $y_1y_{11} \quad 2y_5y_{11}; y_4y_{10} \quad y_2y_{11} \quad y_4y_{11}; y_5y_{10}$;
$y_3y_{11}; y_6y_{10} \quad y_1y_{12} + y_2y_{13} \quad y_4y_{13}; y_7y$ [illegible] $y_{13}; y_8y_{10} \quad y_2y_{13} \quad y_4y_{13}; y_9y_{10} \quad y_3y_{13}$ 10 y

$I_F = h y_{11}^4 \quad y_1 y_5^2 y_{17} + 4 y_3 y_5^2 y_{17} \quad 6 y_5^3 y_{17};\ 4 y_5^2 y_{18} y_{20} + y_{11}^3 \quad y_4 y_5 y_{15};\ y_1 y_{11}^2 \quad 2 y_5 y_{11}^2 \quad y_1 y_4 y_{13} + 4 y_2 y_5 y_{13}$

$2 y_4 y_5 y_{13};\ y_2 y_{11}^2 \quad y_1 y_5 y_{13} + 3 y_3 y_5 y_{13} \quad 4 y_5^2 y_{13};\ y_3 y_{11}^2 + 2 y_5 y_{11}^2 \quad y_2 y_5 y_{13} + y_4 y_5 y_{13};\ y_4 y_{11}^2 \quad y_3 y_5 y_{13} +$

$2 y_5^2 y_{13};\ y_{11}^2 y_{20} \quad y_3 y_5 + 2 y_5^2;\ y_1 y_{16} y_{20} \quad y_{11}^2 \quad y_2 y_{12};\ y_2 y_{16} y_{20} \quad y_1 y_{13} + y_3 y_{13} \quad 2 y_5 y_{13};\ 2 y_1 y_{17} y_{20} + y_{11}^2$

$2 y_2 y_{13} + y_4 y_{13};\ y_2 y_{17} y_{20} \quad y_3 y_{13} + y_5 y_{13};\ 2 y_3 y_{17} y_{20} \quad y_{11}^2 \quad y_4 y_{13};\ y_4 y_{17} y_{20} \quad y_5 y_{13};\ 4 y_5 y_{17} y_{20} + y_{11}^2 \quad y_4 y_{13};$

$y_1 y_{18} y_{20} \quad 2 y_5 y_{18} y_{20} \quad y_2 y_{15} + y_4 y_{15};\ y_2 y_{18} y_{20} \quad y_3 y_{15} + y_5 y_{15};\ y_3 y_{18} y_{20} + 2 y_5 y_{18} y_{20} \quad y_4 y_{15};\ y_4 y_{18} y_{20}$

$y_5 y_{15};\ y_1 y_{19} y_{20} \quad y_2 y_{16} \quad y_2 y_{17} + y_4 y_{17};\ y_7 y_{19} y_{20} \quad y_2 y_{18} \quad 2 y_4 y_{18};\ y_2^2 \quad y_1 y_3 + y_3 y_5 \quad 2 y_5^2;\ y_2 y_3 \quad y_1 y_4 +$

$y_2 y_5 \quad y_4 y_5;\ y_3^2 \quad y_1 y_5 \quad 2 y_5^2;\ y_2 y_4 \quad y_1 y_5 \quad y_3 y_5;\ y_3 y_4 \quad y_2 y_5 \quad y_4 y_5;\ y_4^2 \quad y_3 y_5 \quad 2 y_5^2;\ y_2 y_7 \quad y_1 y_9 \quad y_4 y_{11};$

$y_3 y_7 \quad y_1 y_{11} \quad y_3 y_{11} \quad 2 y_5 y_{11};\ y_4 y_7 \quad y_2 y_{11} \quad 2 y_4 y_{11};\ y_5 y_7 \quad y_3 y_{11} \quad y_5 y_{11};\ y_7^2 \quad y_{11}^2 \quad y_2 y_{12} + y_4 y_{13};\ y_2 y_9$

$y_1 y_{11} \quad y_3 y_{11};\ y_3 y_9 \quad y_2 y_{11} \quad y_4 y_{11};\ y_4 y_9 \quad y_3 y_{11} \quad 2 y_5 y_{11};\ y_5 y_9 \quad y_4 y_{11};\ y_7 y_9 \quad y_1 y_{13} + y_3 y_{13};\ y_9^2 \quad y_2 y_{13}$

$+ y_4 y_{13};\ y_7 y_{11} + y_{11}^2 \quad y_2 y_{13} + y_4 y_{13};\ y_9 y_{11} \quad y_3 y_{13} + 2 y_5 y_{13};\ y_3 y_{12} \quad y_1 y_{13} + y_3 y_{13} \quad 2 y_5 y_{13};\ y_4 y_{12} \quad y_2 y_{13};$

$y_5 y_{12} \quad y_3 y_{13} + y_5 y_{13};\ y_7 y_{12} \quad y_1 y_{15} \quad y_5 y_{15};\ y_9 y_{12} \quad y_2 y_{15};\ y_{11} y_{12} \quad y_3 y_{15} + y_5 y_{15};\ y_{12}^2 \quad y_1 y_{16} + y_3 y_{17}$

$2 y_5 y_{17};\ y_7 y_{13} \quad y_3 y_{15} \quad y_5 y_{15};\ y_9 y_{13} \quad y_4 y_{15};\ y_{11} y_{13} \quad y_5 y_{15};\ y_{12} y_{13} \quad y_1 y_{17} \quad y_3 y_{17};\ y_{13}^2 \quad y_3 y_{17} \quad 2 y_5 y_{17};$

$y_7 y_{15} \quad y_2 y_{16} + 2 y_4 y_{17};\ y_9 y_{15} \quad y_1 y_{17} + 2 y_5 y_{17};\ y_{11} y_{15} \quad y_2 y_{17} + y_4 y_{17};\ y_{12} y_{15} \quad y_1 y_{18} \quad y_3 y_{18};\ y_{13} y_{15}$

$y_3 y_{18} \quad 2 y_5 y_{18};\ y_{15}^2 \quad y_1 y_{20} + 2 y_5 y_{20};\ y_3 y_{16} \quad y_1 y_{17} \quad 2 y_5 y_{17};\ y_4 y_{16} \quad y_2 y_{17} \quad y_4 y_{17};\ y_5 y_{16}$ 2 $y_5 y_{17}$ ... $y_7 y_{16}$ ... y_5

$y_1 y_{18} \quad y_3 y_{18} \quad 2 y_5 y_{18};\ y_9 y_{16} \quad y_2 y_{18} \quad y_4 y_{18};\ y_{11} y_{16} \quad y_3 y_{18};\ y_{12} y_{16};\ y_1 y_{19} + y_2 y_{20} \quad y_4 y_{20};\ y_{13} y_{16} \quad y_2 y_{20}$

$y_4 y_{20};\ y_{15} y_{16} \quad y_7 y_{19} + y_9 y_{20};\ y_{16}^2 \quad y_1 \quad 2 y_5;\ y_7 y_{17} \quad y_3 y_{18} \quad y_5 y_{18};\ y_9 y_{17} \quad y_4 y_{18};\ y_{11} y_{17} \quad y_5 y_{18};\ y_{12} y_{17}$

$y_2 y_{20};\ y$ 25 y 2 $_{20}$; 5 y_9 ... Ro9.967Td [(Ro9.967Td [(R9 Td [(y)8.4809]TJ /R27 6.97385 Tf 4.8 -1.TJ)-15.602 ...

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