DEPARTMENT OF MATHEMATICS

TECHNICAL REPORT

The Importance of, and the Need for, Research on How Students Read and Use their Mathematics Textbook

ANNIE SELDEN
MARY D. SHEPHERD

JUNE 2013

No. 2013-3

TENNESSEE TECHNOLOGICAL UNIVERSITY
Cookeville, TN 38505

The Importance of, and the Need for, Research on How Students Read and Use their Mathematics Textbooks

Annie Selden[1], New Mexico State University
and
Mary D. Shepherd[2], Northwest Missouri State University

Abstract: In this article, we argue that: (1) there has been little research on how students read their mathematics textbooks; (2) there has been research on opportunities to learn from textbooks, on how teachers use textbooks, and on how textbooks are selected; (3) reading comprehension research, while valuable in general, does not sufficiently inform one about good reading strategies for mathematical text; (3) text relevance research on the kinds of goals students have when reading their mathematics textbooks may be a useful direction; and (4) more generally, that research on what parts of their mathematics textbooks students read and use, and why, would greatly inform both research and practice.

In the reform N Q R Z Q D V W K H ³1H[W &DQRQ´ Begle (1973, p. 209) wrote, ³ , I D mathematical topic is in the text[book], then students do learn it. If the topic is not in the text[book], then, on average, students do not learn it.´ More recently, it has been argued that textbooks play an L P S R U W D Q W U R O H L Q X Q L Y H ëñ ñ $" ~•âçÅî¿A/ EB!W ´

(Poisson, 2011). Thus, it would seem that gaining mathematical knowledge from reading, D Q G X V L Q J R Q H ¶ V mathematics textbook is an essential component of learning mathematics both at school and university. But how much do we really know about how students read, and use, their mathematics textbooks?

---

[1] Annie Selden is Professor of Mathematics Emerita from Tennessee Technological University and Adjunct Professor of Mathematics at New Mexico State University.
[2] Mary D. Shepherd is Associate Professor of Mathematics at Northwest Missouri State University.

Prior r esearch on mathematics textbooks

Prior research has often focused on mathematics textbooks themselves, such as on opportunities to learn from them, on how teachers use them, and on how they are selected. For example, researchers have examined textbooks for opportunities to learn such important skills as reasoning and proving (Stylianides, 2009; Thompson, Senk, & Johnson, 2012). Researchers have considered the readability of mathematical textbooks,

instruction? Also, if students attempt to do so, how effective is the advice for doing their homework, passing their tests, or for gaining genuine mathematical understanding?

Special features of mathematical

mathematical reading? Indeed, mathematics textbooks are not alone in having special features.

In 2010, a special section of *Science*, the journal of the American Association for the Advancement of Science (AAAS), was devoted to research on, and to the challenges of, reading the academic language of science. It was noted that, while school students have mastered the reading of various kinds of English text (mostly narratives), this does not suffice for science texts that are precise and concise, avoid redundancy, use sophisticated words and complex grammatical constructions, and have a high density of information-bearing words (Snow, 2010, p. 450). These are some of the same features of mathematical text that seem to make reading such text hard for students.

This special section of *Science* also indicated that collaborations [3] between designers of science curricula and literacy scholars are needed to develop and evaluate methods for helping students master the language of science at the undergraduate and high-school levels as well as at the middle-VFKRRO OHYHO ´ S

Reading comprehension research

6KRXOGQ¶W UHDGLQJ FRPSUHKHQVLRQ UHVHDUFK JLY on how students read, and use, their mathematics textbooks? Not necessarily. For example, such research KDV SURGXFHG IRUPXODV RQ WKH UHDGDELOLW\ RI WH[W The large majority of these formulas are inapplicable to mathematical text because they have no ability to evaluate symbolic notation, much less graphical information. ´ (Horney, 2012). Indeed, while such research can inform us about whether textbooks are written at a level that students should be able to comprehend, it cannot inform us on how students actually read, and use, their mathematics textbooks.

Not only do reading comprehension researchers not seem to investigate the reading of academic text very much, but they also seems to implicitly define good reading as what good readers do, where good student readers are often those who perform better academically. For example, in their meta-analysis of the reading comprehension research literature, Pressley and Afflerbach (1995) examined 38 studies whose readers included 6th graders, high school students, undergraduates, graduate students, PhDs, professors and other professionals. All were good readers in that they had no decoding difficulties and many read texts with which they had a great deal of experience. Pressley and Afflerbach found a great number of activities that good readers do before, during, and after reading. They reduced these to D ³7KXPEQDLO 6NHWFK´ RI 15 Constructively Responsive Reading (CRR) activities, including: (a) preview text before reading to gain an overview: (b) look for and pay attention to important material; (c) activate and use prior knowledge to interpret text; (d) make inferences about information not explicitly stated; (e) determine meanings of new or unfamiliar words1l02 0 1 181.25 488.3 D(a)4(t3u cou60e)-15(8a 62

Ideas from t

Working Memory Capacity

,W KDV EHHQ VKRZQ WKDW ³D UHDGHU¶V ZRUNLQJ PHPRU processing when the \ UHDG IRU GLIIHUHQW SXUSRVHV « Furthermore, working memory resources are important because they also allow the reader to integrate ideas across sentences. ´(Linderholm, Kwon, & Wang, 2011, p. 201). In addition, low WMC readers engage in less effective strategies when reading for study purposes (rather than entertainment purposes), as compared to their high WMC counterparts, and that they also recall less text information (as judged by multiple choice tests), but are overconfident in their performance -- WKH\ EHOLHYH ³WKH\ DUH HQJDJHG LQ VWUDW egies to meet their goals, but IDOO VKRUW ´ /LQGHUKROP .ZRQ :DQJ S

$OWKRXJK ZH KDG LQIRUPDWLRQ RQ RXU VWXGHQWV¶ mathematics scores (Shepherd, Selden, & Selden, 2012), we did not have information on their working memory capacities,[3] so it is possible that, in future studies, knowledge of VWXGHQWV¶ working memory capacities might give additional insights into their reading. Future research studies might consider administering a test for WMC to investigate wheth HU DQG KRZ WKLV DIIHFWV VWXGHQWV¶ UHDGLQJ RI

Standards of coherence

Standards of coherence refer to the types and strength of coherence that an individual reader aims to maintain during reading. These can be implicit or explicit and reflect that LQGLYLGXDO¶V GHVLUHG OHYHO RI XQGHUVWDQGLQJ IRU Standards of

[3] One common test for WMC is the reading span task (RST) invented by Daneman and Carpenter (1980). The original RST required participants to read series of unconnected sentences aloud and to remember the final word of each sentence of a series. The number of sentences of a series was incrementally increased until a participant's reading span, or the maximum number of final words correctly recalled, was found.

coherence are influenced not only by characteristics of the reader, but also by the characteristics of the text.

Characteristics of the text LQFOXGH WKH ³VSHFLILF FRQWHQW RI W which the content is presented, gaps in the semantic flow, layout, [and] the presence of text signals such a V WLWOHV DQG LWDOLFV´ (Bock, et al., 2011, p. 125). It would be useful to investigate which special features of mathematical text JLYH VWXGHQWV¶ GLIILFXOW how one might help students overcome such difficulties.

The characteristics of the reader include working memory capacity, mentioned above, and inadequate or insufficient prior knowledge. ³$ UHDGHU¶V VWDQGDUGV R coherence when reading a particular text are influenced by situation-specific factors such

SUREOHPV >KRPHZRUN H[HUFLVHV@ DQG ,¶OO FR
XQGHUVWDQG DQG ,¶OO « JR EDFN WR WKH VHFWLRQ
saying ...

Additional studies are needed to gain more LQVLJKW LQWR VWXGHQWV¶ VWDQ inference when reading their beginning university mathematics textbooks. But, what about students at other levels? In a study (Shepherd, 2012) of PDWKHPDWLFV JUDGXDWH VWXGH PDWKHPDWLFLDQV¶ UHDGLQJ RI XQIDPLOLDU mathematical text, it was observed that, when a portion of the text was unclear to them, they sometimes read ahead in the hopes that further reading would clarify things. For example, one mathematics graduate student, when reading about a homeomorphism, being defined between two open sets, said,

³$OWKRXJK , GRQ¶W FRPSOHWHO\ XQGHUVWDQG ZKHU
KRZ LW¶V XVHIXO $QG ,¶OO NHHS UHDGLQJ« , ZLOO ´

One can see from the above, that using the concepts of text relevance could be useful for designing studies on, and analyzing data collected on, what, how, and why VWXGHQWV¶ UHDG WKHLU PDWKHPDWLFV WH[WERRNV WKL

What students do when reading their mathematics textbooks.

In addition to our own study (Shepherd, Selden, & Selden, 2012), described above, we know of two other recent studies that have either looked at, or have asked students about, their reading of their mathematics textbooks. Rezat (2010) looked at the actual reading of high achieving German 6th grade and 12th grade students. Instead of using an interview setting and having the students read aloud, he asked students to highlight every part of the textbook that they used, both in and out of class, and to explain in a small booklet why they had used the highlighted parts. In addition, he observed the classroom mathematics lessons and took field note for a period of three weeks. That way he was able to tell whether students only used the textbook when told to by the teacher or whether they also used it of their own accord. They did both.

Rezat found students used their textbooks for solving tasks, for consolidating of mathematical knowledge and skills, for acquiring new mathematical content, and for pursuing their own mathematical interests. However, Rezat also FRPPHQWHG WKDW ³+DU ever does it seem like students want to understand the mathematics first and then apply it WR WKH WDVN´ This seems to agree with our undergraduates (Shepherd, Selden, & Selden, 2012) who stated, in the debrief, that they normally did not read the exposition at the beginning of a section and had only done so during the interview because they were asked to. Rezat indicated the need for more research, stating that ³D EHWWHU XQGHUVWD RI VWXGHQW¶V >VLF@ XWLOL]DWLRQV RI PDWKHPDWLFV V LPSOHPHQWDWLRQ RI PDWKHPDWLFV WH[WERRNV LQWR W

Weinberg, Wiesner, Benesh, and Boester (2012) surveyed 1146 undergraduate students in introductory mathematics classes, such as college algebra, precalculus, discrete mathematics, calculus, and introductory statistics, about what parts of their the VWXGHQWV XVHG WKHLU WH[WERRNV ³PD\ EH WKH UHVX ZHOO DV VWXGHQWV¶ EHOLHIV DERXW UHDGLQJ DQG WKH But these conjectures need further research.

DISCUSSION

It has been noted by Lithner (2004), in his study of the kinds of reasoning required to work calculus textbook tasks, that "it is possible in about 70% of the exercises to base the solution not only on searching for similar situations, but on searching only the solved examples." 7KLV WRJHWKHU ZLWK VRPH RI RXU VWXGHQ reading the textbook exposition (Shepherd, Selden, & Selden, 2012) and the findings of Weinberg, Wiesner, Benesh, and Boester (2012), suggests that an optimally efficient reading strategy for beginning university mathematics students, who are primarily interested in completing their end-of-section homework assignments in order to get good grades, is to look for similar worked examples. Furthermore, for this goal, such students may correctly see it as a waste of time to read the entirety of the preceding textbook section before attempting their homework. Perhaps university instructors and school teachers need to ask more conceptual and integrative questions on assignments and tests in order to encourage students to read the exposition. But this also needs more research.

The ineffective reading we observed in our students (Shepherd, Selden, & Selden, 2012) had to do with not being able to consistently correctly work straightforward tasks, immediately after reading how to work them. It could be that our students were simply not accustomed to reading their textbooks in order to find out how to work tasks, but rather depended greatly on their instructor to illustrate such methods during class. Since the instructor of these students had provided instruction on reading their textbooks

(Shepherd, 2005)

Begle, E. (1973). Some lessons learned by SMSG. *The Mathematics Teacher*, *66*(3), 207-214.

Bock, van den, P., Bohn-Goettler, C. M., Kenedou, P., Carlson S., & White, M. J. (2011). When a reader meets a text: The role of standards of coherence in reading comprehension. In M. T. McCrudden, J. P. Magliano, & G. Schraw (Eds.), *Text relevance and learning from text* (pp. 123-139). Charlotte, NC: Information Age Publishing, Inc.

Daneman, M., & Carpenter, P. A. (1980). Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behavior*, *19*(4), 450-466.