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A PARTIAL CHARACTERIZATION OF THE COCIRCUITS OF A SPLITTING MATROID

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ABSTRACT. This paper describes some of the cocircuits of a splitting matroid $M_{x,y}$ in terms of the cocircuits of the original matroid M.

1. Introduction

The matroid notation and terminology used here will follow Oxley [2]. In particular, the ground set and the collections of independent sets, bases, and circuits of a matroid M will be denoted by $E(M)$, $I(M)$, $B(M)$, and $C(M)$, respectively. The fundamental circuit of an element e with respect to the basis B (see [2, p. 18]) will be denoted by $C(e, B)$.

Fleischner [1] introduced the idea of splitting a vertex of degree at least three in a connected graph and used the operation to characterize Eulerian graphs. For example, the graph G in Figure 1 is obtained from G by splitting away the edges x and from the vertex . Raghunathan, Shikare, and Waphare [3] extended the splitting operation from graphs to binary matroids. One of their results [3, Theorem 2.2] can be used to define the splitting operation in a binary matroid in terms of circuits.

Definition 1.1. Let M be a binary matroid and suppose $x,$ $E(M)$. The splitting matroid M is the matroid having collection of circuits $C(M\ ) =$ C_0 C_1 where
$C_0 = \{C$ $C(M) \mid x,$ C or $x,$ $C\}$; and
$C_1 = \{C_1$ $C_2 \mid C_1, C_2$ $C(M), C_1 \cap C_2 =$ $, x$ $C_1,$ C_2; and there is no C C_0 such that C C_1 $C_2\}$.

The next result, due to Shikare and Asadi [4], characterizes the bases of a splitting matroid M in terms of the bases of the original matroid M.

Lemma 1.2. *Let M be a binary matroid and suppose $x,$ $E(M)$. Then $B(M\ ) = \{B$ $\{\ \} \mid B$ $B(M),$ $E - B$ and the uni ue ir uit ontained in B ontains either x or $\}$.*

The results in the next section describe some of the cocircuits of M in terms of the cocircuits of M. Recall that the cocircuits of a matroid M

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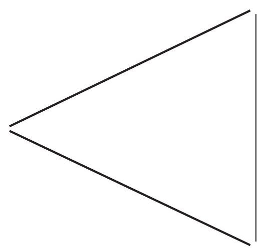

Lemma 2.3. *uppose* C

So $C(\ , B)$ contains neither x nor and it follows that B $B(M\ )$. We conclude that $C' - \{x,\ \}$. Hence $(B\ ) \cap (C' - \{x,\ \}) =$. Therefore each basis of M must have non-empty intersection with $C' - \{x,\ \}$.

We now show that $C' - \{x,\ \}$ is a minimal set having non-empty intersection with all bases of M . Let B be a basis of M so that x B, B, and $B \cap (C' - \{x,\ \}) =$. Let $C' - \{x,\ \}$. If $C(\ , B)$ does not contain x, then $|C(\ , B) \cap C'| = 1$; a contradiction. Thus x $C(\ , B)$. Moreover, $C(\ , B)$ and it follows that B $B(M\ )$. Since for all $C' - \{x,\ \}$, the set B is a basis of M , the set $C' - \{x,\ \}$ is minimal having non-empty intersection with each basis of M . We conclude that $C' - \{x,\ \}$ is a cocircuit of M . □

Cocircuits of M	Type I sets	Cocircuits of M
$\{\ ,\ \}$	$\{\ \}$	$\{\ ,\ \}$
$\{\ ,\ ,\ \}$	$\{\ ,\ \}$	$\{\ \}$
$\{\ ,x,\ \}$		$\{\ ,\ \}$
$\{\ ,x,\ ,\ \}$		$\{x,\ \}$
$\{\ ,\ ,e,\ \}$		$\{\ ,\ ,e,\ \}$
$\{\ ,\ ,e,\ \}$		$\{\ ,x,e,\ \}$
$\{\ ,x,e,\ \}$		$\{\ ,\ ,e,\ \}$
$\{\ ,x,e,\ \}$		$\{\ ,x,e,\ \}$
$\{\ ,x,e,\ ,\ \}$		$\{\ ,x,e,\ \}$
$\{\ ,x,e,\ ,\ \}$		$\{\ ,\ ,e,\ \}$
$\{\ ,\ ,\ ,e,\ \}$		$\{\ ,\ ,e,\ \}$
$\{\ ,\ ,\ ,e,\ \}$		$\{\ ,x,e,\ \}$

Notice that the cocircuits of M are $\{x,\ \}$, the Type I sets of M, the sets $\ -\ $ for each cocircuit $\ $ of M containing a Type I set , and the cocircuits of M that do not contain a Type I set. The following conjecture proposes that this relationship holds in general.

Conjecture 2.5. *uppose the sp ittin matroid M is obtained rom M and $\{x,\ \}$ is a proper subset o a o ir uit o M. Then*

$$C(M\ ) = \begin{cases} \{x,\ \} \\ C - \{x,\ \} \text{ or ea h o ir uit } C \text{ o } M \text{ proper y ontainin } \{x,\ \} \\ \ -\ \text{ or ea h o ir uit } \ \text{ o } M \text{ ontainin a Type set} \\ C \text{ o } M \text{ su h that } C \text{ does not ontain a Type set} \end{cases}$$

acknowledgement

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