DEPARTMENT OF MATHEMATICS

TECHNICAL REPORT

CONSCIOUSNESS IN MATHEMATICAL PROBLEM SOLVING: THE FOCUS, THE FRINGE, AND NON-SENSORY PERCEPTION

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In this paper, we will discuss the way various features of consciousness interact with each other and with cognition, specifically, the cognition of mathematical reasoning and problem solving. Thus we are interested in how consciousness and cognition "work," in a somewhat mechanistic way, rather than in larger philosophical questions about consciousness. Our goal is ultimately to answer questions like: Where does one's "next" idea come from? Answers to such smaller questions may eventually help in understanding the nature of consciousness itself.

We will discuss the relationship between consciousness and cognition in terms of two illustrations, and recall and extend some features of consciousness pointed out by

2. We see the development of non-sensory experiences as a form of learning and as possibly "linked" to other remembered expe

up, then one would very likely not be able to continue. Furthermore, between steps one is likely not to be conscious of anything happening. Clearly, however, something is happening (outside of consciousness) because the steps are not at all random.

We think that, for someone knowledgeable in algebra, the "decisions" needed to guide writing the various steps are made outside of consciousness and based on earlier steps that have become conscious. Furthermore, such "decisions" are ephemeral, and thus cannot be the basis for further "decisions," unless the earlier "decisions" are acted upon in a way that becomes conscious. Finally, the information needed to make such "decisions" seems to be very durable, always available, and need not become conscious to be usable. For example, that information might include: "It's OK to combine the $4x$ and the $7x$ on the right."

student-generated purported proofs of a single theorem (one right and three wrong) were only 46% correct, i.e., they might as well have flipped coins. However, in describing their previous experience in reading (correct textbook) proofs, the students sounded competent and emphasized reading for "understanding" (Selden & Selden, 2003).

When both mathematicians and students come to the end of a purported proof "something" tells them either the proof is correct or they should reexamine it for errors. For mathematicians, we suggest this "something" is either a *feeling of correctness* (which differs from a feeling of understanding, by including logical correctness of the proof) or a *feeling of caution*. In contrast, the students we studied appeared to be using their *feeling of understanding*, which served them poorly.

The validation of proofs is *not* often explicitly taught, but perhaps ought to be. We hope this analysis exposes an important pedagogical question: How does one teach a feeling?

In 1980 Rosnick and Clement introduced the Students-and-Professors Problem:

> *Write an equation using the variables S and P to represent the following statement: "There are six times as many students as there are professors at this university." Use S for the number of students and P for the number of professors.*

It turned out that many subjects (~40% of freshman engineering students), who "ought" to be able to correctly so145 eat tcts4messors.the Students danger of being set up incorrectly. Thus, they could not recognize the Students-and-Professors Problem as such a problem. They would have nothing to link to, or generate, a feeling of caution, and hence, would not experience it and have no reason to check their initial attempt at writing an equation.

Of course, this still does not answer the deeper psychological question of why there are algebra problems that are in danger of being set up incorrectly.

We have studied the ability of university calculus students to solve five *moderately nonroutine problems*, that is, problems moderately similar to, but not exactly like, problems they had been taught to solve (Selden, Selden, Hauk, & Mason, 2000; Selden, Selden, & Mason, 1994; Selden, Mason, & Selden, 1989). Such problems are important because there is no way to teach all, or even most, problems that can occur in the real world. Very few of even the most successful students could solve even one of the five problems, and taking additional calculus/differential equations classes helped only a little. Furthermore, often students who did not solve a problem could be seen, in a subsequent test, to have had *adequate knowledge* to solve it. For these students, the appropriate knowledge apparently was not lacking, but *did not come to mind*. Such students do not seem to think of various ways to begin a solution.

In contrast, calculus teachers do not have this difficulty. If they ue[o3 Tw([l)-1.98the46(sTD0.00

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