
DEPARTMENT OF MATHEMATICS
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A GENERALIZATION OF A GRAPH RESULT OF HALIN AND JUNG

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A GENERALIZATION OF A GRAPH RESULT OF HALIN AND JUNG

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Abstract. This paper provides a partial generalization to matroid theory of the result of Halin and Jung that each simple graph with minimum vertex degree at least 4 has K_5 or the octahedron $K_{2,2,2}$ as a minor.

1. Introduction

Let G be a simple graph with minimum vertex degree at least 4. Then G has K_5 or the octahedron $K_{2,2,2}$ as a minor.

$$A_{10} \left[\begin{array}{c|c} I_5 & \\ \hline & \end{array} \right] \quad A_{12} \left[\begin{array}{c|c} I_6 & \\ \hline & \end{array} \right]$$

Figure 1. $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} .

Lemma 2.2. *If M is a 3-connected band algebra and M is an F_7^* -module, then $M \cong e_a M \cong M \sim F_7$.*

Proof. Let M be a 3-connected band algebra and M be an F_7^* -module. Then M is a module over $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} . Let A_{10} and A_{12} be the matrices in Figure 1. Then M is a module over $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} . \square

Lemma 2.3. *Let e be an idempotent in R_{10} . Then $R_{10}/e \sim M^*(K_{3,3})$.*

Lemma 2.4. *Let M be a 3-connected band algebra. Then $e_e M \cong a_c c \cong a_c c$, M is a module over R_{10} and R_{12} .*

Proof. Let M be a 3-connected band algebra. Then M is a module over $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} . Let A_{10} and A_{12} be the matrices in Figure 1. Then M is a module over $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} . Let e be an idempotent in R_{10} . Then $R_{10}/e \sim M^*(K_{3,3})$. Let G be a module over $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} . Then $G \sim K_{3,3}$. Let M be a module over $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} . Then $M \sim M^*(K_5)$. Let M be a module over $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} . Then $M \sim M^*(K_5)$. Let M be a module over $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} . Then $M \sim M^*(K_5)$. Let M be a module over $GF(\mathbb{F}_3)$ -modules R_{10} and R_{12} . Then $M \sim M^*(K_5)$. \square

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