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DEPARTMENT OF MATHEMATICS  
TECHNICAL REPORT

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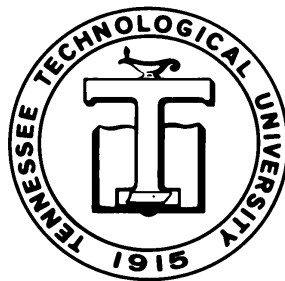
VARIOUS ESTIMATIONS OF  $\pi$   
AS DEMONSTRATIONS OF  
THE MONTE CARLO METHOD

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Figure 3.1: A diagram showing a circle of radius  $\theta$  inscribed within a square of side length  $2\theta$ . The square is divided into four quadrants by a horizontal and a vertical line passing through the center of the circle. The area of the circle is labeled  $A_c$  and the area of the square is labeled  $A_s$ . The radius  $\theta$  is indicated by a dimension line from the center to the top edge of the square.

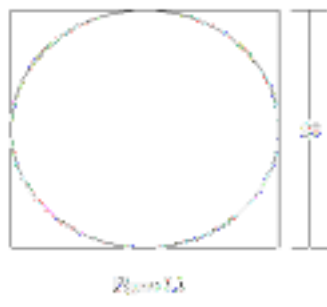


Figure 3.2: A diagram showing a circle of radius  $\theta$  inscribed within a square of side length  $2\theta$ . The square is divided into four quadrants by a horizontal and a vertical line passing through the center of the circle. The area of the circle is labeled  $A_c$  and the area of the square is labeled  $A_s$ . The radius  $\theta$  is indicated by a dimension line from the center to the top edge of the square.

$$A_c/A_s = \frac{\pi\theta^2}{4\theta^2}$$

$$\Rightarrow \pi = 4(A_c/A_s)$$

Figure 3.2: A diagram showing a circle of radius  $\theta$  inscribed within a square of side length  $2\theta$ . The square is divided into four quadrants by a horizontal and a vertical line passing through the center of the circle. The area of the circle is labeled  $A_c$  and the area of the square is labeled  $A_s$ . The radius  $\theta$  is indicated by a dimension line from the center to the top edge of the square.

$$\pi \approx 4(N_c/A_s) \quad (3.2) \quad B=30.$$

Table 3.2

$N_c$	100	1000	10000	100000	1000000	10000000
$E$	3.14	3.144	3.1359	3.14160	3.141690	3.1416010
$A$	0.01	0.003	0.0001	$3.0 \times 10^{-5}$	$5.0 \times 10^{-6}$	$2.0 \times 10^{-7}$

2.  $\frac{d}{dt} \left[ \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} k x^2 + \frac{1}{2} k y^2 \right] = 0$   
 3.  $\ddot{x} + \omega^2 x = 0$ ,  $\ddot{y} + \omega^2 y = 0$   
 4.  $x(t) = A \cos(\omega t + \phi)$ ,  $y(t) = B \cos(\omega t + \psi)$

$I_n$  是  $n$  阶贝塞尔函数的积分， $A$  是常数。

$$I_n \approx \sqrt{2\pi}$$

当  $n$  很大时， $I_n$  的渐近展开式为：

$$\pi \approx \frac{I_n^2}{2}$$

由此可得  $\pi$  的近似值。

### 3.3

$n$	100	1000	10000	100000	1000000	10000000
E	4.0	3.5	3.15	3.143	3.1398	3.14206
A	2.0	0.2	0.02	0.002	0.0002	$2.0 \times 10^{-5}$