DEPARTMENT OF MATHEMATICS
TECHNICAL REPORT

PERFECT BINARY MATROIDS

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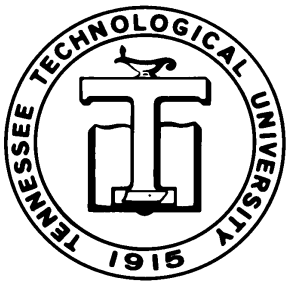

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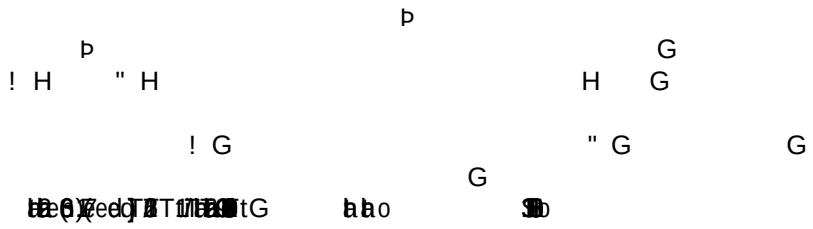
PERFECT BINARY MATROIDS

ALLAN D. MILLS

Abstract. In this paper a definition of perfect binary matroids is considered and it is shown that, analogous to the Perfect Graph Theorem of Lovász and Fulkerson, the complement of a perfect matroid is also a perfect matroid. In addition, the classes of critically imperfect graphic matroids and critically imperfect graphs are compared.

Introduction



G Þ {k G k }
M

p M

Definition 1.2.
GF q Þ c M q {j ∈ p M q
M

j p M j >
G k "G k >

Lemma 1.3. c M q {j ∈ N p M q k□

Perfect Binary Matroids

Þ

Definition 2.1. GF q M ! M|F q
c M|F q ß F M

! M c M ! M c M

Example 2.2. ! PG n – , c PG n – , n ß
PG n – ,

Example 2.3. M K_4 " K_4
c M K_4 $K_{4\ M\ K}$

GF q

$! M_2$ $c M_2$

$$\{! M_1, ! M_2\} \qquad \{c M_1, c M_2\}$$

$$c N$$

$$\geq \; ! N$$

$$\geq \; \{! M_1, ! M_2\}.$$

$c N$ $! N$

F $ß$ N F $F_1 \cup F_2$ F_1 $ß$ M_1 F_2 $ß$ M_2 $! M_1|F_1$ $c M_1|F_1$ $! M_2|F_2$ $c M_2|F_2$ $N|F$ $M_1|F_1 \oplus M_2|F_2$ $\{c M_1|F_1, c M_2|F_2\}$ $c N|F$

Critically Imperfect Graphs and Matroids

G

Definition 3.1. M M
M|F ß F M

Example 3.2. C_n n n
C_n c M C_n C_n
! M C_n c M C_n |F ! M C_n |F
ß F M C_n

G

P G ,

c M G

G ∼ K_5

K_5 G

" G^0

" $\overline{C_n}$ ≥

G^0

G

" G^0 G^0

K_4 G^0

" G^0

$\overline{C_7}$

□

M $\overline{C_7}$ $\overline{C_7}$

$\overline{C_7}$