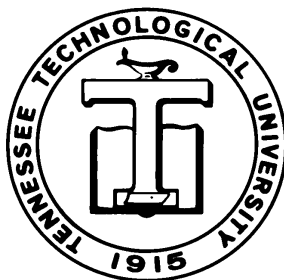

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ON THE SEPARATING IDEALS OF SOME
VECTOR-VALUED GROUP ALGEBRAS

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Abstract. For a locally compact Abelian group G , and a commutative Banach algebra B , let $L(G, B)$

see [5,8,9,12].

2. PRELIMINARIES. Let B be a commutative Banach algebra (not necessarily unital), and let G be a locally compact Abelian group with Haar measure m . Throughout the following, the dual group of G is denoted by \hat{G} and the spectrum of B is denoted by $\sigma(B)$.

(B). Let $L^1(G, B)$ denote the Banach algebra of all integrable function from G into B ,

$$(f * g)(t) := \int_G f(s)g(st) \, m(s)$$

(i) $f \circ x \in L(G, B)$, and $\|f \circ x\| = \|f\| \|x\|$

(ii) $(f \pm g) \circ x = f \circ x \pm g \circ x$

(iii) $\widehat{f \circ x} = \widehat{f} \circ x$

(iv) $(f \circ x) \circ (g \circ x) = (f \circ g) \circ xy$

(v) If B

i , and $\|g\| = 1$. We have

$$\begin{aligned}
 & \|f - \sum_i x_i (f_i - x_i) - g \hat{f}(!)\| \\
 &= \|f - \sum_i x_i (h_i - x_i) + \sum_i x_i (h_i - x_i) - \sum_i x_i \underline{(f_i - x_i)} - g \hat{f}(!)\| \\
 & \leq \|f - \sum_i x_i (h_i
 \end{aligned}$$

Proof. By Lemma 3.1, there exist f_1, f_2, \dots, f_n in $L(G)$ with compactly supported Fourier transforms, and x_1, x_2, \dots, x_n in B such that

$$\|f - \sum_{i=1}^n f_i * x_i\| < \frac{\epsilon}{2} + \|\hat{f}(\cdot)\|$$

where $\hat{f}_i(\cdot) = 0$. Since $L(G)$

to \mathcal{P} . For, if g belongs to $L(G)$ with $\hat{g}(\lambda) \neq 0$, $\hat{g} = 0$ on V , and x a non-zero vector in B , then $(g * x)^\wedge = \hat{g}(\lambda) \hat{x}(\lambda)$ (the zero vector of $L(G, B)$). Since \mathcal{P} is a prime ideal of $L(G, B)$, either $g * x \in \mathcal{P}$ or $\hat{x} \in \mathcal{P}$. But $\hat{g}(\lambda) \hat{x}(\lambda) \neq 0$. Hence $\hat{x} \in \mathcal{P}$. Thus all the functions f in $L(G, B)$ with vanishing Fourier transforms in a neighborhood of λ belong to \mathcal{P} . Hence by Lemma 3.2, it follows that \mathcal{P} is dense in M_γ . This completes the proof of the theorem. \square

Theorem 3.5. Let G be a noncompact locally compact Abelian group, and B be a commutative Banach algebra. If \mathcal{P} is a closed prime ideal of $L(G, B)$ contained in $M_{\gamma, \phi}$ for some $\lambda \in \hat{G}$, and $\phi \in \hat{B}$, then \mathcal{P} contains M_γ . Furthermore \mathcal{P} does not contain M_σ for any $\sigma \neq \lambda$.

Proof. Let $f \in M_\gamma$. By Corollary 3.3, f can be approximated by a function g in $L(G, B)$ with vanishing Fourier transform in a neighborhood V of λ . By an argument similar to the one given in Theorem 3.4, we can show $g \in \mathcal{P}$. Since \mathcal{P} is a closed ideal, it follows that $f \in \mathcal{P}$. Thus M_γ is contained in \mathcal{P} . Let $\sigma \in \hat{G}$ such that $\sigma \neq \lambda$. Suppose V_σ and V_γ are compact neighborhoods of σ and λ respectively such that $V_\sigma \cap V_\gamma = \emptyset$. Then there exist functions f_σ and f_γ from G into the complex plane with the support of \hat{f}_σ contained in V_σ and the support of \hat{f}_γ contained in V_γ such that $\hat{f}_\sigma(\sigma) = 1$ and $\hat{f}_\gamma(\lambda) = 1$. Let $x, y \in B$ such that $\phi(x)\phi(y) \neq 0$. Then $f_\sigma * x, f_\gamma * y \in L(G, B)$ such that $(f_\sigma * x)^\wedge(\sigma) = \phi(x)$ and $(f_\gamma * y)^\wedge(\lambda) = \phi(y)$. Since \mathcal{P} is a prime ideal contained in $M_{\gamma, \phi}$, we get $f_\sigma * x \in \mathcal{P}$. Obviously $f_\gamma * y \notin \mathcal{P}$.

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epimorphism from a commutative Banach algebra X onto A , let $\mathfrak{S}(h) =: \{a \in A \mid \text{there is a sequence } (x_n) \text{ in } X \text{ with } x_n \rightarrow 0 \text{ and } h(x_n) \rightarrow a\}$. It is easy to show that $\mathfrak{S}(D)$ and $\mathfrak{S}(h)$ are closed subspaces. The closed graph theorem D is continuous if and only if $\mathfrak{S}(D)$ is zero. Similarly, h is continuous if and only if $\mathfrak{S}(h)$ is zero. It is well known that $\mathfrak{S}(D)$ and $\mathfrak{S}(h)$ are submodules of A (see [1]). For further information on these ideals, their relation to the theory of automatic continuity theory, see [2].

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some $! \#$. Let $\mathcal{M}_\Delta = \{! \# \mid M_\gamma + \mathcal{P} \text{ for some } \mathcal{P} \# \mathcal{M}_\Delta\}$. Obviously \mathcal{M}_Δ is a finite set. Since \mathfrak{S} is contained in ah

of containing $!$. Since G

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