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ON THE SEPARATING IDEALS OF SOME VECTOR-VALUED GROUP ALGEBRAS

Ramesh Garimella

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TENNESSEE TECHNOLOGICAL UNIVERSITY
Cookeville, TN 38505

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RAMESH V. GARIMELLA

Abstract. For a locally compact Abelian group G, and a commutative Banach algebra B, let L (G, B

see [5,8,9,12].

2. PRELIMINARIES. Let B be a commutative Banach algebra (not necessarily unital), and let G be a locally compact Abelian group with Haar measure m. Throughout the following, the dual group of G is denoted by   and the spectrum of B is denoted by (B). Let L (G, B) denote the Banach algebra of all integrable function from G into B,

$$(f\ !\ g)(t) := \int_G$$

(i) f % x # L (G, B), and ‖f % x‖ = ‖f‖ ‖x‖

(ii) (f ± g) % x = f % x ± g % x

(iii) $\hat{f}$ % x(!) = $\hat{f}$(!)x

(iv) (f % x) ! (g % x) = (f ! g) % xy

(v) If B

i, and $\|g\| = 1$. We have

$$\|f \;" \sum_i (f_i \,\%\, x_i) \;"\; g \,\%\, \hat{f}(!)\|$$

$$= \|f \;" \sum_i (h_i \,\%\, x_i) + \sum_i (h_i \,\%\, x_i) \;" \sum_i (f_i \,\%\, x_i) \;"\; g \,\%\, \hat{f}(!)\|$$

$$' \;\|f \;" \sum_i (h_i$$

Proof. By Lemma 3.1, there exist f , f , . . . , f_n in L (G) with compactly supported Fourier transforms, and x , x , . . . , x_n in B such that

$$\|f \text{ " } \sum_i f_i \,\%\, x_i\| < \frac{^2}{2} + \|\hat{f}(!)\|$$

where $\hat{f}_i(!) = 0$. Since L (G

to $\mathcal{P}$. For, if g belongs to L (G) with ĝ(!) ≠ 0, ĝ = 0 on " V , and x a non-zero vector in B, then (g % x) ! f = (the zero vector of L (G, B)). Since $\mathcal{P}$ is a prime ideal of L (G, B), either g % x # $\mathcal{P}$ or f # $\mathcal{P}$. But g %̂ x(!) = ĝ(!)x ≠ ". Hence f # $\mathcal{P}$. Thus all the functions f in L (G, B) with vanishing Fourier transforms in a neighborhood of ! belong to $\mathcal{P}$. Hence by Lemma 3.2, it follows that $\mathcal{P}$ is dense in M_γ. This completes the proof of the theorem. ¥

Theorem 3.5. Let G be a noncompact locally compact Abelian group, and B be a commutative Banach algebra. If $\mathcal{P}$ is a closed prime ideal of L (G, B) contained in $M_{\gamma,\phi}$ for some ! # , and # # (B), then $\mathcal{P}$ contains M_γ. Furthermore $\mathcal{P}$ does not contain M_σ for any ' ≠ !.

Proof. Let f # M_γ. By Corollary 3.3, f can be approximated by a function g in L (G, B) with vanishing Fourier transform in a neighborhood V of !. By an argument similar to the one given in Theorem 3.4, we can show g # $\mathcal{P}$. Since $\mathcal{P}$ is a closed ideal, it follows that f # $\mathcal{P}$. Thus M_γ is contained in $\mathcal{P}$. Let ' # such that ' ≠ !. Suppose V_σ and V_γ are compact neighborhoods of ' and ! respectively such V_σ \$ V_γ =). Then there exist functions f_σ and f_γ from G into the complex plane with the support of $\hat{f}_\sigma$ contained in V_σ and the support of $\hat{f}_\gamma$ contained in V_γ such that $\hat{f}_\sigma$(') = 1 and $\hat{f}_\gamma$(!) = 1. Let x, y # B such that #(x)#(y) ≠ 0. Then f_σ % x, f_γ % y # L (G, B) such that (f_σ % x) ! (f_σ % y) = . Since $\mathcal{P}$ is a prime ideal contained in $M_{\gamma,\sigma}$, we get f_σ % x # $\mathcal{P}$. Obviously f_γ % y #̸ $\mathcal{P}$.

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epimorphism [redacted]tive Banach algebra X onto A, let ℑ(h) =: {a # A| there is a sequence {[redacted]& 0 and h(x_n) & a}. It is easy to show that ℑ(D),and ℑ(h) are closed [redacted] closed graph theorem D is continuous if and only [redacted] ℑ(D) is zero. Sim[redacted] if and only if ℑ(h) is zero. It is well known[redacted] ℑ(D) and ℑ(h) are s[redacted]13]). For further information o[redacted] ideals, their relation to th[redacted] automatic continuity theory, see[redacted]

Now we are ready to state one of the ma[redacted].53 4D78atemzeroa(su.

some ! # . Let $_{M\,\triangle}$ = {! # |M_γ + $\mathcal{P}$ for some $\mathcal{P}$ # $\mathcal{M}_\triangle$}. Obviously $_{M\,\triangle}$ is a finite set. Since $\Im$ is contained in ah of containing ! . Since G

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Department of Mathematics
Tennessee Technological University
Cookeville, TN 38505 USA
e-mail: RGarimella@tntech.edu